Theoretical background.

Methodology and instrumentation equipment.

The HVSR (Horizontal to Vertical Spectral Ratio) technique consists in the direct measurement of the fundamental frequencies of buildings and of the subsoil, using the ambient noise (microtremors), aimed at estimating of seismic effects and of vulnerability of buildings. Ambient noise is composed by all the vibrations propagating in the subsoil, originated by both natural and anthropic phenomenons and modeled as a wavefield generated by an array of independent point-like sources. From 1970's it was recognised that microtremors tend to excite the natural frequencies of the subsoil, allowing to identify them. By a practical point of view, the instrument measures the velocities of the microtremors along both the horizontal (H) and vertical (V) planes and the ratio between the two components (H/V), inside a selected interval of frequencies, usually from 0.1 to 100 Hz. Local maximum peaks of H/V, at which correspond local minimums of V, identify the frequencies of resonance of the subsoil below the vertical of measurement. A higher amplitude of H/V peaks indicates a higher contrast between the seismic impedance of contiguous layers, which means a higher difference of velocity.

To perform a HVSR measurement, it needs to use a triaxial velocimeter, which allows to record the microtremors along two orthogonal directions (X and Y) in the horizontal plane and along the vertical one (Z), using a wide interval of frequencies (0.1-100Hz) and for a duration long enough (10-20 minutes). The subsoil motion, due to the passage of the waves, is measured as velocity, using the sampling step imposed by the operator. The recorded measurements are processed and then displayed in graphic format as HV spectra (H/V ratio as a function of frequency, where H is the average between X and Y) and V spectra (motion component along Z).

HVSR technique allows:

- ❑ to evaluate the local seismic effects, the site amplification and the soil liquefaction risk;
- ❑ to get a profile of the shear waves as a function of the depth;
- ❑ to analyze the seismic vulnerability of buildings.

Local seismic effect.

Introduction.

Shear waves (S) are the main cause of building damages during a seismic event. In fact, while the compression waves (P) act along the vertical direction, the shear waves strain structures along the horizontal vector, where buildings are more vulnerable. Thus in seismic risk analyses is fundamental to examine the way of propagation of S waves. It's in fact widely demonstrated that this type of oscillation, during the path from the bedrock to the ground surface, can be subject to a filtering, which tends to redistribute the energy of the wave train, concentrating it into specific frequencies, corresponding to the natural frequencies of vibration of the subsoil. As a whole it could have an amplification effect of the S waves, which will act on the building. This phenomenon can arise due to topographic peculiarities of the site (topographic amplification), like buried valleys or slopes, or to sudden changes of the mechanical behavior of the subsoil with the depth.
Lermo and Chavez-Garcia (1993), after Nakamura (1989), suggest that the HV spectrum could be interpreted as representative of the transfer function of the seismic motion from the bedrock to the ground surface. Thus, in accord to these authors, the amplitudes of the HV peaks could be interpreted directly as amplification factors of the seismic motion, at least respect to the stratigraphic component. It has to take into account that Authors consider, as sources of microtremors, earthquakes of low energy, with ipocentre deep in the crust. In this contest the HV spectrum can be considered formed essentially by P and S waves. In the HVSR technique, commonly used in applied geology, the recorded microtremors derive from sources positioned on the ground surface and they're composed essentially by surface waves (Rayleigh and Love waves). In this case, while it's still possible to link the HV maximum peaks to the resonance frequencies of the subsoil, the correspondent amplitudes can't be interpreted as amplification factors anymore. Consequently, to evaluate the real transfer function of the subsoil, it needs to use the Vs profile gotten by the inversion of the HV spectrum and then to calculate the theoretical curve through an analytical or numerical method.

1D propagation model of S waves into a horizontal layered subsoil.

Vertical propagation of shear waves, with frequency ω, causes horizontal displacements u(z,t), which must satisfy the equation:

$$(1)\ \rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial z^2} + \eta \frac{\partial^3 u}{\partial z^2 \partial t}$$

Harmonic displacements u with frequency ω, can be expressed in the following form too:

$$(2)\ u(z,t) = U(z)e^{i\omega t}$$

Substituting (2) into (1):

$$(3)\ (G + i\omega\eta)\frac{\partial^2 U}{\partial z^2} = \rho\omega^2 U$$

where ρ is the mass density and G is the shear modulus of the soil layer.
Equation (3) has the following general solution:

$$(4)\ U(z) = Ee^{ikz} + Fe^{-ikz}$$

where:

$$k = \sqrt{\frac{\rho\omega^2}{G^*}}$$

G* is the complex shear modulus:

$$G^* = G(1 + 2i\beta)$$

and β is the critical damping factor:

$$\beta = \frac{\omega\eta}{2G}$$

In equation (4) E is the incident wave traveling upwards and F is the reflected wave traveling downwards.
Combine (2) and (4):

$$(5)\, u(z,t) = \left(Ee^{ikz} + Fe^{-ikz}\right)e^{i\omega t}$$

In a multilayer subsoil, at the top of the layer *n,* with thickness *h,* the displacements are:

$$(6)\ u_n(z=0) = \left(E_n + F_n\right)e^{i\omega t}$$

at the bottom:

$$(7)\ u_n(z=h) = \left(E_n e^{ik_n h_n} + F_n e^{-ik_n h_n}\right)e^{i\omega t}$$

Shear stresses acting along the horizontal plane at the top and bottom of the layer are respectively:

$$(8)\, \tau_n(z=0) = ik_n G_n^{*}\left(E_n + F_n\right)e^{i\omega t}$$

$$(9)\, \tau_n(z=h) = ik_n G_n^{*}\left(E_n e^{ik_n h_n} + F_n e^{-ik_n h_n}\right)e^{i\omega t}$$

Shear strains ($\gamma(z,t) = \frac{\partial u}{\partial t}$) are instead:

$$\gamma_n(z=0) = ik_n\ \left(E_n + F_n\right)e^{i\omega t}$$
$$\gamma_n(z=h) = ik_n\left(E_n e^{ik_n h_n} + F_n e^{-ik_n h_n}\right)e^{i\omega t}$$

In a multilayer subsoil the parameters ρ, G e β change generally with the depth. Thus E and F have different values. Stresses and displacements must be continuous at all the interfaces. Amplitudes E and F of the incident and reflected wave of the layer *n* can be expressed in the following way:

$$(10)\; E_{n+1} = \frac{1}{2}E_n(1+\alpha_n)e^{ik_n h_n} + \frac{1}{2}F_n(1-\alpha_n)e^{-ik_n h_n}$$

$$(11)\; F_{n+1} = \frac{1}{2}E_n(1-\alpha_n)e^{ik_n h_n} + \frac{1}{2}F_n(1+\alpha_n)e^{-ik_n h_n}$$

where:

$$\alpha_n = \sqrt{\frac{\rho_n G_n^{*}}{\rho_{n+1} G_{n+1}^{*}}}$$

is the complex impedance ratio.
At the ground surface the shear stress must be zero. From equation (8):

$$E_1=F_1.$$

For the case $E_1=F_1=1$, the amplitudes E and F of the layer *n* can be determined by substituting this condition in the formulas (10) and (11) , starting from the ground surface to the bedrock.
Transfer function between the displacements at level *n* and *n+1* is defined by:

$$(12)\; A_{n+1,n}(\omega) = \frac{u_n}{u_{n+1}} = \frac{E_n + F_n}{E_{n+1} + F_{n+1}}$$

At the bedrock interface E’=F’ (shear stress=0). As a result transfer function of the shear wave at the ground surface in respect to the bedrock is given by:

$$(13)\; A_{bedrock,1}(\omega) = \frac{1}{E_{bedrock}}$$

Soil liquefaction hazard.

Nakamura (1996) proposed to use the K_g parameter gotten by the HV technique to evaluate the seismic vulnerability of sites, particularly in respect to the soil liquefaction hazard. K_g, vulnerability index of site, is gotten by the following formula:

$$K_g = \frac{A_g^{\,2}}{F_g}$$

where A_g is the maximum amplitude of the HV spectrum and F_g the corresponding frequency. Sites with a K_g index higher than 10 can be associated to a substantial vulnerability and thus to a significant soil liquefaction hazard, if there are obviously the geological conditions so that the phenomenon may occur.

Evaluating the Vs profile.

The calculation of the Vs profile can be performed through the conditioned inversion of the HV spectrum, using the formula which associates the resonance frequency of the subsoil (f) to the velocity of the shear waves (Vs) :

$$f(Hz) = \frac{Vs}{4h}$$

where h is the depth of the bottom of the soil layer. If the depth, or the S waves velocity, of a soil layer is known, usually the first from the ground surface, it's possible to perform the inversion of the HV spectrum, with the aim to find the best-fitting theoretical curve.

The inversion procedure involves the choose of an initial stratigraphic model and the following calculation of the associated HV spectrum. The theoretical curve is compared to the experimental one and, through an iterative procedure, the initial model is modified by trial to obtain the best overlapping between the curves. The theoretical spectrum is usually calculated through the procedure suggested by Arai e Tokimatsu (2004):

$$(H/V)(f) = \sqrt{\frac{P_{HR} + P_{HL}}{P_{VR}}}$$

where:

P_{HR} = spectral amplitude of the horizontal component of the Rayleigh waves;

P_{HL} =spectral amplitude of the horizontal component of the Love waves;

P_{VR} = spectral amplitude of the vertical component of the Rayleigh waves.

The vertical amplitude of the Love waves is null.

The parameters P_{HR}, P_{HL} and P_{VR} are evaluated with the propagator matrix method, ideated by Thomson (1950) and Haskell (1953) and reformulated by Dunkin (1965) and Watson (1970).

To take into account the weakly dissipative behavior of the subsoil, the velocity values of the P and S waves inserted in the calculation are corrected through a damping factor. The program has default values of the damping factor equal to 0.05 for the S waves and to 0.017 for the P waves.

Peaks at higher frequencies (>10 Hz) indicate the presence of stratigraphic bounds close to the ground surface, vice versa peaks a lower frequencies (<1 Hz) are associated at very deep layers. Since the velocity inversions, that is the decrease of the velocities of the S and P waves with the depth, don't produce peaks inside the HV spectrum, these usually can't be directly identified. An indication of the presence of a velocity inversion is gotten indirectly by the observation of wide intervals inside the HV spectrum, where the HV ratio is below the one value. It's possible to input velocity inversions in the initial model provided that the negative variations are not too sudden.

Seismic vulnerability of buildings.

The seismic vulnerability of buildings is the susceptibility of the structures to be subjected to damages due the seismic force during an earthquake. In recent buildings, the seismic vulnerability is only marginally associated to the edification techniques. Much more important is the seismic amplification of the ground motion, which tends to increase the entity of the seismic force acting against the building. Particularly, a resonance frequency of the building similar to the subsoil one involves the arising of the coupled

resonance, which causes a great increase of the seismic force.
The resonance frequency of buildings can be evaluated through empirical formulas, like the following:

$$fs = \frac{1}{C_1 Z^{\frac{3}{4}}}$$

where C_1 is a variable factor, depending on the edification technique and Z is the height of the building.

Description	C_1
Steel-frame buildings	0,085
Reinforced concrete-frame buildings	0,075
Others	0,050

The formula is applicable only in case of building in which its Z is lower than 40 meters and its mass is distributed uniformly.
In case of existent building the value of the resonance frequency can be measured directly using a triaxial velocimeter. Practically, it has to record the ratio between H_1 and H_0, where H_0 is the spectrum of the horizontal component, X or Y, measured on the first floor, and H_1 is the same component measured on the last floor. The maximum positive peak of the H_1/H_0 spectrum identifies the fundamental resonance frequency of the building.

Geotechnical parameters.

- ❑ LOW STRAIN PARAMETERS.

SHEAR MODULUS.

$$G(kPa) = \rho V_s^2$$

where:

ρ (kNs^2/m^4) = mass density = unit weight / g (9.81 m/s^2);

V_s (m/s) = S wave velocity.

BULK MODULUS.

$$M(kPa) = \rho\left(V_p^{\,2} - \frac{4}{3}V_s^{\,2}\right)$$

where:

V_p (m/s) = P wave velocity.

OEDOMETRIC MODULUS.

$$E_{ed}(kPa) = \rho V_p^{\,2}$$

YOUNG MODULUS

$$E(kPa) = 2\rho V^{s}{}_{2}\left(1+\nu\right)$$

where:

ν = Poisson's ratio, given by:

$$\nu = \frac{\left[0.5\left(\frac{V_p}{V_s}\right)^2 - 1\right]}{\left(\frac{V_p}{V_s}\right)^2 - 1}$$

- ❑ HIGH STRAIN PARAMETERS

YOUNG MODULUS

E = 0.1877 Ey (Fahey & Carter, 1993):

where:

Ey(MPa) = low strain modulus.

PEAK ANGLE OF INTERNAL FRICTION

$$\varphi(°) = 3.9{V_{s1}}^{0.44}$$ (Uzielli et al., 2013)

where: $V_{s1} = \frac{V_s}{\left(\frac{\sigma_v'}{\sigma_{atm}}\right)}$, and σ_v'= vertical effective lithostatic pressure σ_{atm} = atmospheric pressure = 9.81 kPa

UNDRAINED COHESION

$$c_u(kPa) = \left(\frac{V_s}{7.93}\right)^{1.59}$$ (Levesques et al., 2007),

R.Q.D.

$$RQD\% \approx 100\left(\frac{Vs}{Vs_{lab}}\right)^2$$

where:

Vs_{sito} = S wave velocity misured in the site;
Vs_{lab} = S wave velocity of the intact rock.