

# Theoretical basis

## 1.1 Preface

The aim of the geophysical survey based on the down-hole technic consists in the directly evaluation of the propagation velocities inside the subsoil of both compression (P) and shear (S) waves and in the indirectly assessment, by the means of the velocity values obtained, of the mechanical properties of the investigated lithologies.

The down-hole technic involves placing of a vibration source on the ground surface and the measurement of the waves in a borehole. The probe, which contains 1 or 2 3- components geophones (1 vertical and 2 horizontal, orthogonal disposed), are fixed inside a PVC pipe along the borehole. PVC pipe is coupled by the means of cement mortar in the gap between the pipe and the borehole walls. Through a sampling step usually set to 1 m, subsoil stratigraphy is determined.

The vibration source is composed by beating hammer, which in the same time works as a starter, being linked by the means of a trigger to the seismograph. Execution involves a vertical blow, to generate P waves, and two horizontal blows, with pulse direction inverted, to produce S waves.

## 1.2 Interpretation

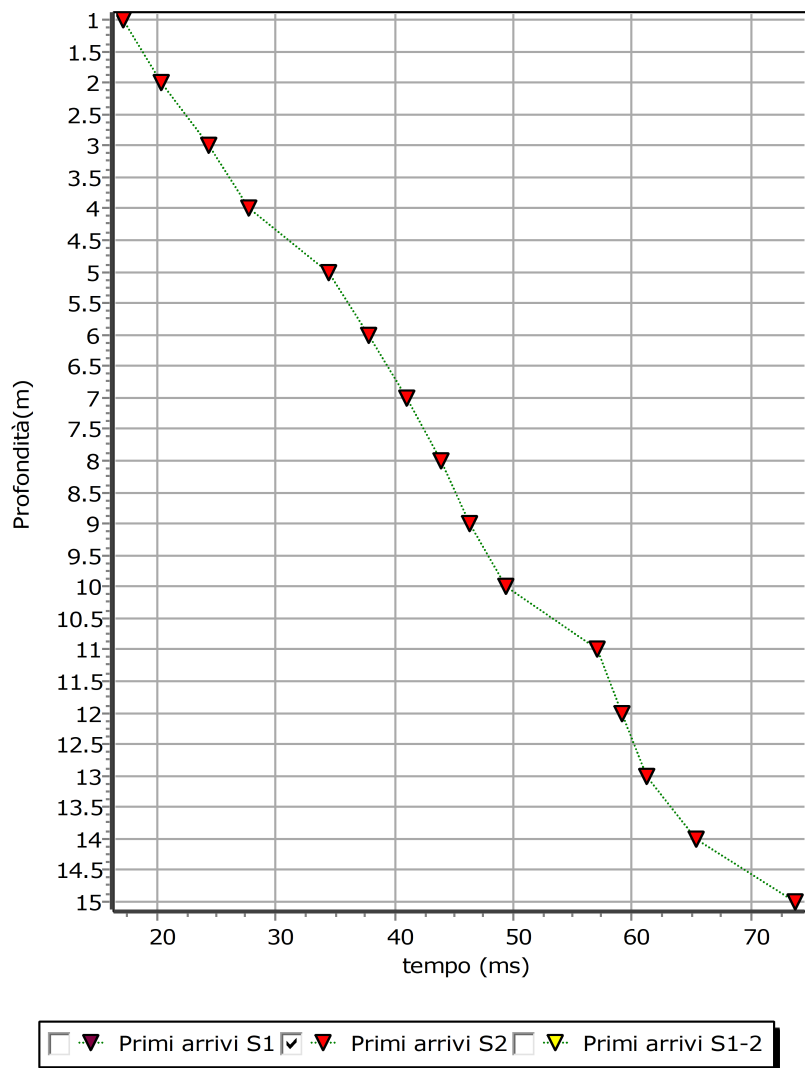
Analysis may be executed both in time and frequency domains.

In case of probe with a single receiver, one can proceed in time domain only. In this condition, first arrivals have to be initially corrected as a function of surface source position. In case of homogeneous soil, correction assumes the following form:

$$t^* = \frac{z}{\sqrt{z^2 + x^2}} t$$

where  $z$  is the receiver depth,  $x$  is the horizontal distance between borehole and source and  $t$  the recorded arrival time.

Conversion from first arrivals to wave velocity ( $V_p$  and  $V_s$ ) is performed, assessing the slope of the straight segment that joins the contiguous points, in the time-depth chart.



Time-depth chart

In case of two-receiver probe, resolution is improved by contemporary recording of the arrivals to the two geophones. Being  $t_1$  and  $t_2$  the first arrivals to the two receivers, after executing time corrections as a function of the source position, propagation velocities in the  $ds$  interval, with  $ds$  = receiver spacing, is given by:

$$V = \Delta s / (t_2^* - t_1^*) = \Delta s / \Delta t^*$$

Calculated velocity  $V$  is named 'interval velocity'.

One may operate in this way in case of single receiver only, imposing  $\Delta s$  = sampling step. In this case  $V$  is named 'pseudo-interval velocity'.

Still operating in time domain,  $dt$  variable may be obtained in a direct way, without executing the signal picking, by the Cross Correlation method. Cross correlation function is calculated

summing the product of the signal amplitudes recorded by the two receivers,  $x(t)$  and  $y(t)$ , progressively shifting one of them of a time interval  $\tau$ , increasing it up to a maximum value equal to the total recording duration:

$$CC_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t) \cdot y(t + \tau) dt$$

where time  $T$ , in which the function reaches its maximum value, represents the time shifting undergone by the signal moving from receiver 1 to receiver 2. Consequently velocity is given by:

$$V = \Delta s / \tau$$

Still in the hypothesis of two receivers, one may execute the interpretation in frequency domain. By the means of the direct Fourier transform (FT), from the two recorded signals the corresponding Fourier spectra can be obtained, both the amplitude and phase ones. The the Cross Power Spectrum (CPS), defined as the product of the Fourier spectrum of the  $y(t)$  signal and the conjugate complex of the Fourier spectrum of the  $x(t)$  signal:

$$CPS_{yx}(f) = Y(f) \cdot X^*(f)$$

CPS vector is characterized by an amplitude  $A_{cps}$  and a phase  $\phi_{cps}$ , respectively given by the product of the amplitudes  $e$  by the difference of the phases of the correspondent Fourier spectra of the signals  $x(t)$  and  $y(t)$ .

Phase velocity  $V(f)$ , as a function of frequency, is given by the following relationship::

$$V(f) = 360^\circ \cdot \frac{\delta_{12}}{\phi_{CPS}(f)} \cdot f$$

where  $\delta_{12}$  is the receiver spacing. The inverse Fourier transform of the CPS vector gives, in a more directly and efficient way, time  $\tau$ , that is the time shifting undergone by the signal moving from receiver 1 to receiver 2. In this case too, velocity is given by:

$$V = \Delta s / \tau$$

### 1.3 Geotechnical parameters

#### □ LOW STRAIN PARAMETERS.

SHEAR MODULUS.

$$G(kPa) = \rho V_s^2$$

where:

$\rho$  (kNs<sup>2</sup>/m<sup>4</sup>) = mass density = unit weight / g (9.81 m/s<sup>2</sup>);  
 $V_s$  (m/s) = S wave velocity.

BULK MODULUS.

$$M(kPa) = \rho \left( V_p^2 - \frac{4}{3} V_s^2 \right)$$

where:

$V_p$  (m/s) = P wave velocity.

OEDOMETRIC MODULUS.

$$E_{ed}(kPa) = \rho V_p^2$$

YOUNG MODULUS

$$E(kPa) = 2\rho V_s^2(1 + \nu)$$

where:

$\nu$  = Poisson's ratio, given by:

$$\nu = \frac{\left[ 0.5 \left( \frac{V_p}{V_s} \right)^2 - 1 \right]}{\left( \frac{V_p}{V_s} \right)^2 - 1}$$

#### □ HIGH STRAIN PARAMETERS

YOUNG MODULUS

$E = 0.1877 E_y$  (Fahey & Carter, 1993):

where:

$E_y$ (MPa) = low strain modulus.

## PEAK ANGLE OF INTERNAL FRICTION

$$\varphi(^{\circ}) = 3.9V_{s1}^{0.44} \quad (\text{Uzielli et al., 2013})$$

where:  $V_{s1} = \frac{V_s}{\left(\frac{\sigma_v'}{\sigma_{atm}}\right)}$ , and  $\sigma_v'$  = vertical effective lithostatic pressure  $\sigma_{atm}$  = atmospheric pressure = 9.81 kPa

## UNDRAINED COHESION

$$c_u (kPa) = \left(\frac{V_s}{7.93}\right)^{1.59} \quad (\text{Levesques et al., 2007}),$$

R.Q.D.

$$RQD\% \approx 100 \left(\frac{V_s}{V_{s_{lab}}}\right)^2$$

where:

$V_{s_{sito}}$  = S wave velocity measured in the site;  
 $V_{s_{lab}}$  = S wave velocity of the intact rock.