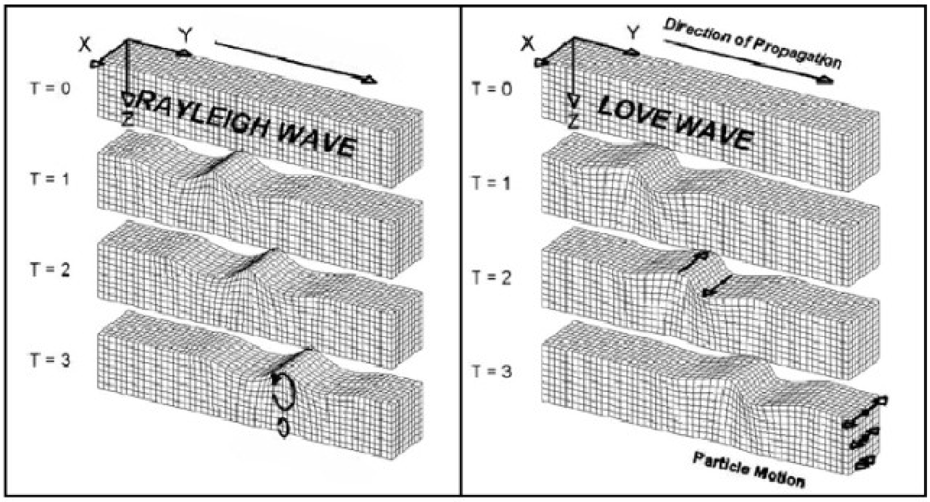
1 Theoretical background.[i]

1.1.Method description.

ReMi technique (Refraction Microtremor) (Louie, 2001) is a method of geophysical survey, developed and spread in the last 15 years, based on the propagation process of surface waves, particularly Rayleigh ones, produced by active sources. Rayleigh waves move along the ground-air surface, assembled by multiple refraction and reflection of compressional waves (P) and shear waves propagating in vertical direction (SV).



Geophysical survey techniques based on the recording and the processing of the ground micro-displacements due to the transit of surface waves (Rayleigh and Love) take advantage of some features which make them different from the body waves (P e S).

1. Energy attenuation rate of surface waves is lesser than P and S ones, thereby they can travel to a longer distance. This because their radiation pattern, generated by a punctiform source, is essentially two-dimensional, whereas, in the case of body waves, it's hemispherical. Thus attenuation rate of surfaces waves as a function of the distance (R) from the source is inversely proportional to $R^{0,5}$, whereas, in the case of body waves, it's inversely proportional to R^2. This more rapid energy attenuation of body

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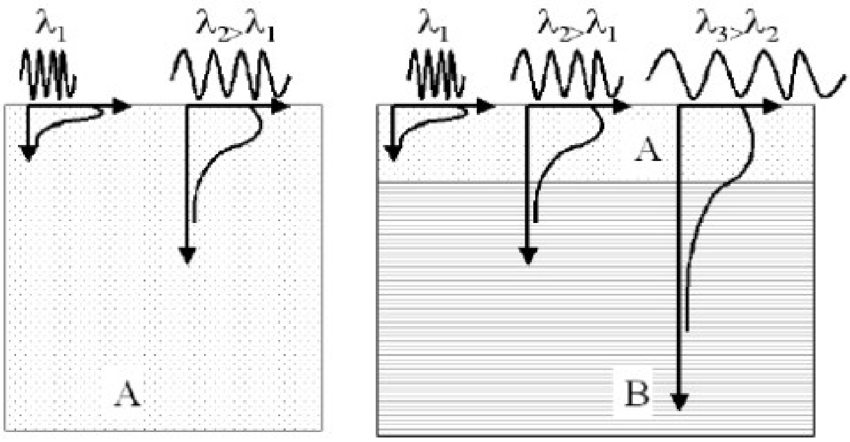
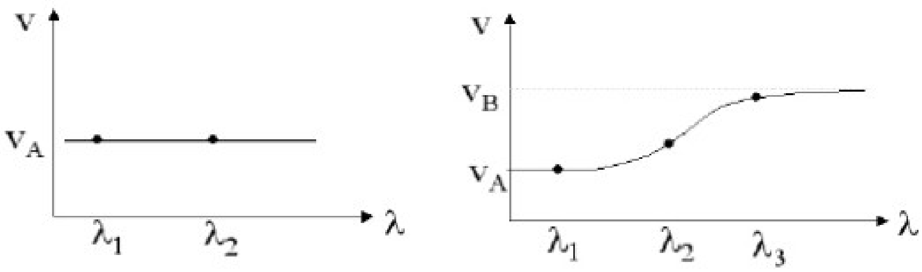
[i] Illustrations by Claudio Strobbia, 2001

waves explains why, in vertically inhomogeneous and normally dissipative media (S wave velocity increases with depth), at a distance next to $\lambda_R/2$ (where λ_R is the Rayleigh wave length taken into account) contribution of body waves gets negligible.

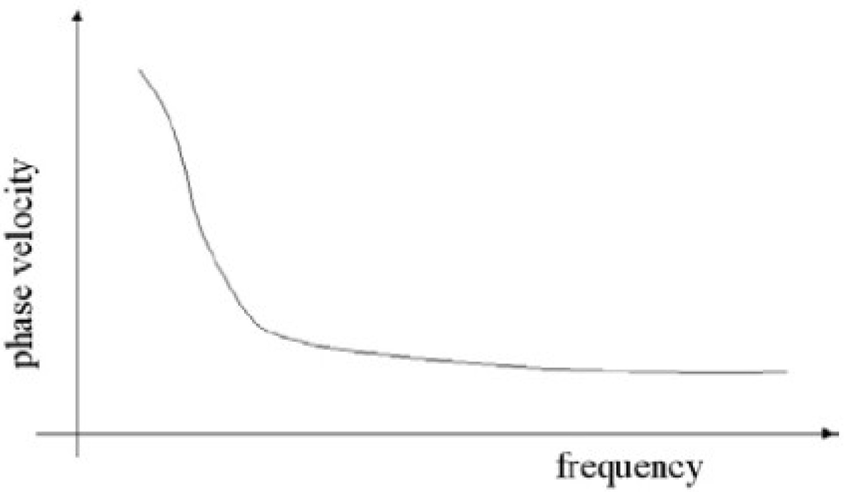
2. Inside an elastic homogeneous medium the maximum depth where the soil is involved by Rayleigh wave propagation is close to λ_R. Inside an elastic inhomogeneous medium this depth is normally between $0.5\lambda_R$ and $1.0\lambda_R$. This means that higher wave lengths (lower frequencies) carry information about the deeper soil layers, while lower ones (higher frequencies) involve the shallower layers.



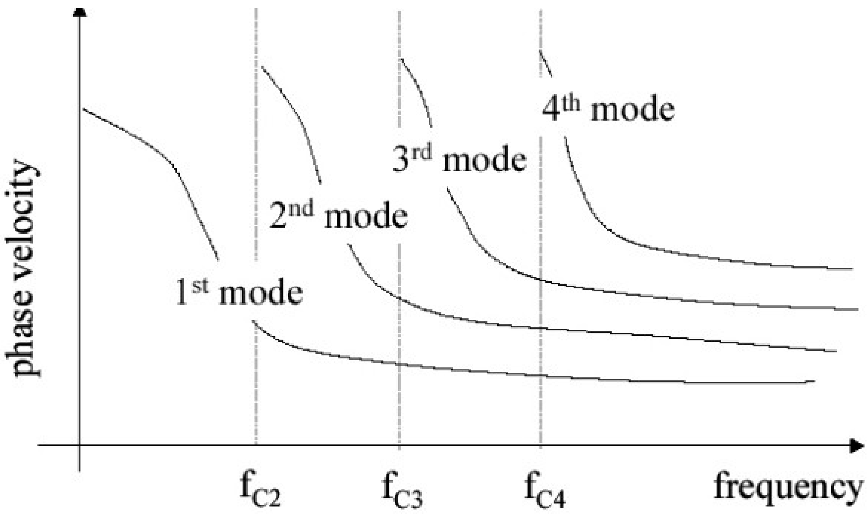
3. Propagation velocity of a surface wave with frequency *f* (or wave length λ) is defined *phase velocity*. Inside a homogeneous medium *phase velocity* (V_A) doesn't change varying the wave frequency. Vice versa into a layered medium, where soil layers with different geomechanical characteristics are stacked, phase velocity (V_B) depends on *f* (or λ).



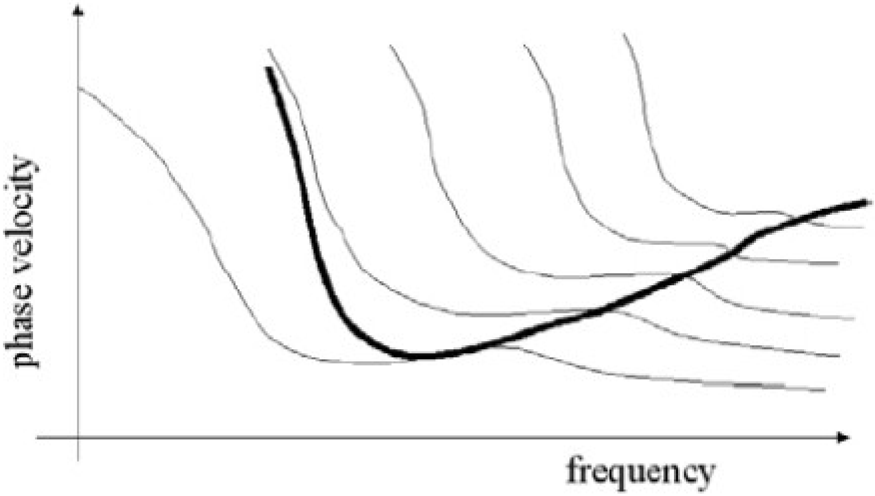
ReMi survey technique takes advantage of these three features of the surface waves, particularly of the Rayleigh ones, to get a profile of the trend of the phase velocity as a function of frequency, aimed at correlating it to the shear wave velocity.
Characteristic (1) allows to make recording of the surface waves generated by the source without a substantial disturbance from body waves. Characteristic (2) permits to assign every frequency of the signal, or wave length, to a different survey depth: lower frequencies are associated to the higher depths and vice versa. Finally, characteristic (3) allows to join every frequency to a specific phase velocity.
The aim of the analysis of a signal recording by ReMi technique is essentially to get the dispersion curve of the phase velocity of the Rayleigh waves. The term ‘dispersion curve’ means that phase velocities tends to disperse themselves , inside an inhomogeneous medium, as a function of frequency.



Inside a homogeneous medium it’s impossible to have a dispersion curve, because phase velocity doesn’t change varying the frequency. This is the cause why ReMi technique cannot work in case of big-thickness homogeneous soil or rock layers.
An important complication arises due to, still in an inhomogeneous medium, the different modes of vibration of the soil, because every mode can be associated to a different dispersion curve.

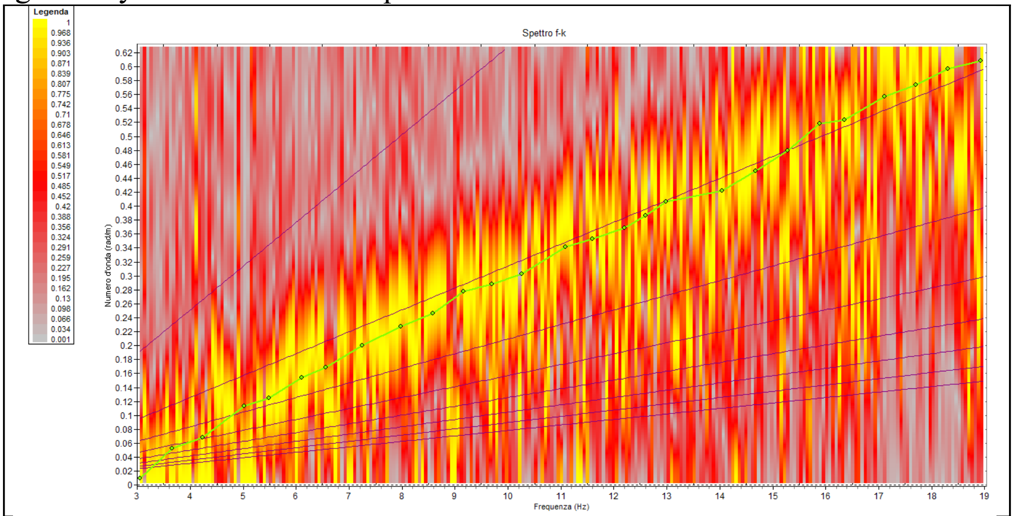


Most of the signal energy is usually associated to the fundamental mode (first mode), but, in some cases, at least inside specific frequency intervals, superior modes can be dominant. It happens, i.e., in the case of substantial inversion of velocity (inversely dispersive soil) or because of low resolution in signal sampling. In these cases joining of the maximum spectral amplitude of phase velocity, for every frequency value, allows to draw an apparent dispersion curve, composed by superimposing of different modes of vibration.

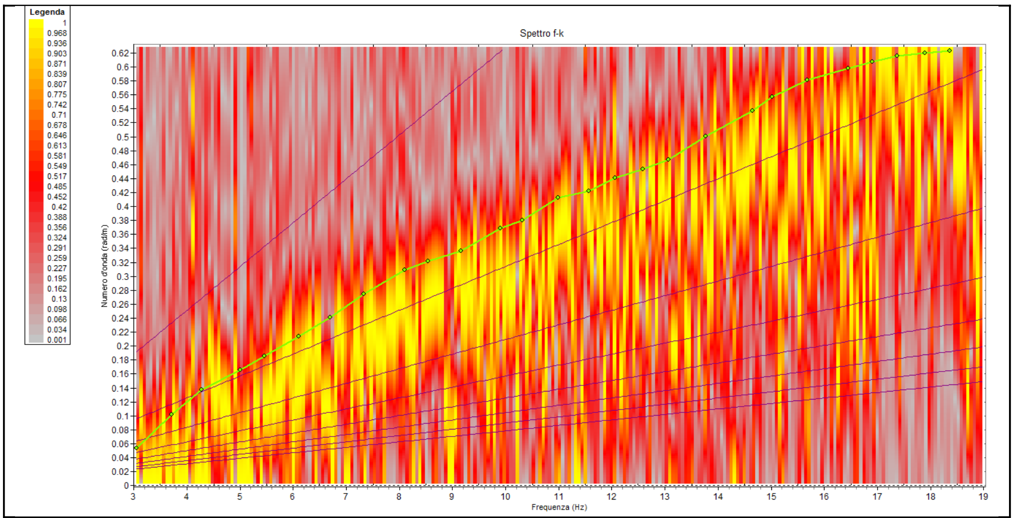


Procedures of inversion requests a link to a specific mode of vibration, generally the first (fundamental), which is usually the easier to identify. Thus it's important don't make mistakes selecting the dispersion curve, merging points actually belong to different modes. This can lead to overestimate V_s, if the inversion is referred to the first mode.
Starting from the recorded seismogram, the frequency-phase velocity graphic is processed by the application of a double Fourier transform to the survey traces, first in the time domain (sampling time-frequency *f*) and then in the space one (receiver spacing-wave number *k*). This process allows to get a graphic, the f-k spectrum, where the distribution of the energy density inside the signal is put in evidence. As just at small distance from the source the contribution of body waves to the signal energy gets negligible, the f-k spectrum allows to highlight the dispersion curve of the surface waves.

Unlike the MASW technique, where the positions of the energy sources in known, being parallel to the line of the receivers, in the case of ReMi technique the disposition of the microtremor sources is generally unknown. When there's a clearly identifiable dominant source, it's preferable to dispose the line of the receivers parallel to it. But in the most cases the source positions isn't easily identifiable and there isn't often a dominant source. The importance of the source position in respect to the line of the receivers is due to the fact that the propagation velocity of the waves is given by the ratio between the spacing of the receivers and the travel time of the wave front. This velocity has its minimum value when the source is parallel to the line of the receivers. In this case the wave front moves in the same direction of the survey line. If the propagation angle, respect to the survey line, isn't zero, the measured velocity will be always higher than the real one, tending to an infinite value when the angle is next to 90°. So, during the interpretation step, it has to select, inside the f-k spectrum, not the zone of the maximum amplitudes, except the case of alignment between the survey line and the sources, but a parallel range where the amplitudes will be lower. To select the correct range is useful superimposing the dispersion curve gotten by the MASW technique.



Selecting the zone of maximum amplitude.



Selecting the zone of 80% respect to the maximum amplitude.

Transition from the f-k spectrum to the frequency-phase velocity graphic is gotten, transforming *k* values into *v* values by the formula:

$$v = 2\,\pi\, f / k$$

Concerning the useful frequency window for data processing, it usually includes a range from 3-4 Hz to 50-60 Hz. Lower limit is bounded to the resonance frequency of the receivers, which is usually 4.5 Hz. Upper limit derives from the consideration that surface waves with frequency over 50-60 Hz carry information about the first centimeters below the ground, thus they're of scarce practical utility.
Furthermore there's an important restriction on the time sampling of the signal, due to existence of a maximum value of frequency, named *Nyquist frequency*, beyond which signals cannot be processing usefully. Nyquist frequency is linked to the sampling step Δt (in seconds) by the formula:

$$f_{Nyquist} = 1\,/2\Delta t$$

Similar limit can be identified for the wave number. There's a maximum value of the wave number, named *Nyquist wave number*, beyond which signals cannot be processing usefully. Nyquist wave number is linked to the receiver spacing Δx by the expression:

$$k_{Nyquist} = \pi / \Delta x$$

Interpretation of the dispersion curve is carried out by a process named *inversion*. In the case of layered soils a direct solution, a solution which permits to transform directly the dispersion curve into a stratigraphic model, cannot exist, because of the not univocity of the problem (from the same curve it's possible to get many different Vs profiles). Consequently it needs to operate, using a reverse process. After selecting an initial stratigraphic model, inversion can be performed by the ordinary least squares method, modifying, through an iterative process, some soil parameters, share wave velocity, soil unit weight, thickness and Poisson's ratio of each model layer, and then recalculating, by each iteration, the standard deviation of the new model compared to the previous one. The zeros of the so-named secular function are calculated by the propagator matrix method, conceived, by the theoretical point of view, by Thomson (1950) and Haskell (1953) and then reformulated by Dunkin (1965) and Watson (1970). To take into account the weakly dissipative behavior of the soil, the velocity values of the body waves, inserted in the model, are modified, applying a damping factor. Inside the program are imposed a 0.05 damping factor for the P waves and a 0.017 damping factor for the S ones.

1.2. Earthquake site effects

1.2.1. Introduction.

Shear waves (S) are the main cause of building damages during a seismic event. In fact, while the compression waves (P) act along the vertical direction, the shear waves strain structures along the horizontal vector, where buildings are more vulnerable. Thus in seismic risk analyses is fundamental to examine the way of propagation of the S waves. It's in fact widely demonstrated that this type of oscillation, during the path from the bedrock to the ground surface, can be subject to a filtering, which tends to redistribute the energy of the wave train, concentrating it into specific frequencies, corresponding to the natural frequencies of vibration of the soil. As a whole it could have an amplification effect of the S waves, which will act on the building. This phenomenon can arise due to topographic peculiarities of the site (topographic amplification), like buried valleys or slopes, or to sudden changes of the mechanical behavior of the subsoil with the depth.

1.2.2. 1D propagation model of S waves into a horizontal layered subsoil.

Vertical propagation of shear waves, with frequency ω, causes horizontal displacements u(z,t), which must satisfy the equation:

$$(1)\, \rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial z^2} + \eta \frac{\partial^3 u}{\partial z^2 \partial t}$$

Harmonic displacements *u* with frequency ω, can be expressed in the following form too:

$$(2)\, u(z,t) = U(z) e^{i\omega t}$$

Substituting (2) into (1):

$$(3)\left(G+i\omega\eta\right)\frac{\partial^2 U}{\partial z^2}=\rho\omega^2 U$$

where ρ is the mass density and G is the shear modulus of the soil layer.
Equation (3) has the following general solution:

$$(4)U(z)=Ee^{ikz}+Fe^{-ikz}$$

where:

$$k=\sqrt{\frac{\rho\omega^2}{G^*}}$$

G* is the complex shear modulus:

$$G^*=G\left(1+2i\beta\right)$$

and β is the critical damping factor:

$$\beta=\frac{\omega\eta}{2G}$$

In equation (4) *E* is the incident wave traveling upwards and *F* is the reflected wave traveling downwards.
Combine (2) and (4):

$$(5)u(z,t)=\left(Ee^{ikz}+Fe^{-ikz}\right)e^{i\omega t}$$

In a multilayer subsoil, at the top of the layer *n,* with thickness *h,* the displacements are:

$$(6)\ u_n(z=0)=\left(E_n+F_n\right)e^{i\omega t}$$

at the bottom:

$$(7)\ u_n(z=h) = \left(E_n e^{ik_n h_n} + F_n e^{-ik_n h_n}\right) e^{i\omega t}$$

Shear stresses acting along the horizontal plane at the top and bottom of the layer are respectively:

$$(8)\ \tau_n(z=0) = ik_n G_n^{*}\left(E_n + F_n\right) e^{i\omega t}$$

$$(9)\ \tau_n(z=h) = ik_n G_n^{*}\left(E_n e^{ik_n h_n} + F_n e^{-ik_n h_n}\right) e^{i\omega t}$$

Shear strains ($\gamma(z,t) = \dfrac{\partial u}{\partial t}$) are instead:

$$\gamma_n(z=0) = ik_n\ \left(E_n + F_n\right) e^{i\omega t}$$
$$\gamma_n(z=h) = ik_n\left(E_n e^{ik_n h_n} + F_n e^{-ik_n h_n}\right) e^{i\omega t}$$

In a multilayer subsoil the parameters ρ, G e β change generally with the depth. Thus E and F have different values. Stresses and displacements must be continuous at all the interfaces. Amplitudes E and F of the incident and reflected wave of the layer *n* can be expressed in the following way:

$$(10)\ E_{n+1} = \frac{1}{2}E_n\left(1+\alpha_n\right)e^{ik_n h_n} + \frac{1}{2}F_n\left(1-\alpha_n\right)e^{-ik_n h_n}$$
$$(11)\ F_{n+1} = \frac{1}{2}E_n\left(1-\alpha_n\right)e^{ik_n h_n} + \frac{1}{2}F_n\left(1+\alpha_n\right)e^{-ik_n h_n}$$

where:

$$\alpha_n = \sqrt{\frac{\rho_n G_n^{*}}{\rho_{n+1} G_{n+1}^{*}}}$$

is the complex impedance ratio.
At the ground surface the shear stress has to be zero. From equation (8):

$$E_1=F_1.$$

For the case $E_1=F_1=1$, the amplitudes E and F of the layer *n* can be determined by substituting this condition in the formulas (10) and (11), starting from the ground surface to the bedrock.
Transfer function between the displacements at level *n* and *n+1* is defined by:

$$(12)\ A_{n+1,n}(\omega)=\frac{u_n}{u_{n+1}}=\frac{E_n+F_n}{E_{n+1}+F_{n+1}}$$

At the bedrock interface E'=F' (shear stress=0). As a result transfer function of the shear wave at the ground surface in respect to the bedrock is given by:

$$(13)\ A_{bedrock,1}(\omega)=\frac{1}{E_{bedrock}}$$

1.3 Geomechanical parameters of the soil layers.

- LOW STRAIN PARAMETERS.

SHEAR MODULUS.

$$G(kPa) = \rho V_s^{\,2}$$

where:
ρ (kNs^2/m^4) = mass density = unit weight / g (9.81 m/s^2);
V_s (m/s) = S wave velocity.

BULK MODULUS.

$$M(kPa) = \rho\left(V_p^{\,2} - \frac{4}{3}V_s^{\,2}\right)$$

where:
V_p (m/s) = P wave velocity.

OEDOMETRIC MODULUS.

$$E_{ed}(kPa) = \rho V_p^{\,2}$$

YOUNG MODULUS

$$E(kPa) = 2\rho V^{s}{}_{2}(1+\nu)$$

where:
ν = Poisson's ratio, given by:

$$\nu = \frac{\left[0.5\left(\frac{V_p}{V_s}\right)^2 - 1\right]}{\left(\frac{V_p}{V_s}\right)^2 - 1}$$

- ❑ HIGH STRAIN PARAMETERS

YOUNG MODULUS

E = 0.1877 Ey (Fahey & Carter, 1993):

where:
Ey(MPa) = low strain modulus.

PEAK ANGLE OF INTERNAL FRICTION

$$\varphi(°) = 3.9 V_{s1}^{\ 0.44}$$ (Uzielli et al., 2013)

where: $V_{s1} = \frac{V_s}{\left(\frac{\sigma_v'}{\sigma_{atm}}\right)}$, and σ_v'= vertical effective lithostatic pressure σ_{atm} = atmospheric pressure = 9.81 kPa

UNDRAINED COHESION

$$c_u(kPa) = \left(\frac{V_s}{7.93}\right)^{1.59}$$ (Levesques et al., 2007),

R.Q.D.

$$RQD\% \approx 100\left(\frac{Vs}{Vs_{lab}}\right)^2$$

where:

Vs_{sito} = S wave velocity misured in the site;
Vs_{lab} = S wave velocity of the intact rock.