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Theoretical fundamentals

1 Bearing capacity of shallow foundations.

1.1 Introduction.

By the term foundation one refers to the structure fitted to transmit the load of the building and other surcharges acting on it to the underground. The global load has not to overtake the maximum shear strength of the soil layers. If this would happen, the foundation will undergo a sudden shear failure associated to wide settlements, not tolerable by the building. The maximum theoretical load that a foundation can support immediately before the failure is termed bearing capacity.

Foundation is defined 'shallow' if the following relation is satisfied:

$$D < 4 \times B;$$

where D is the depth of embedment below the ground surface and B is the width of the foundation (B less than or equal to L, length of the foundation). Otherwise the foundation is defined a deep foundation.

1.2 Bearing capacity through analytical methods

1.2.1 Terzaghi (1943).

The Terzaghi formula has the following form:

$$Q = c \times N_c \times s_c + \gamma_1 \times D \times N_q + 0.5 \times \gamma_2 \times B \times N_y \times s_y;$$

where:

N_c, N_q, N_y = adimensional bearing capacity factors associated, respectively, to the contribute from cohesive layers, from the weight of the soil above the depth of embedment and from granular layers.

Terzaghi suggested the following relationships:

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$$N_q = a^2 / [2 \times \cos^2(45 + \varphi/2)]$$

where $a = \exp[(0.75 \times \pi - \varphi/2) \times \text{tg}(\varphi)]$;

$$N_c = (N_q - 1) \times \text{cotg}(\varphi)$$

$$N_y = [\text{tg}(\varphi)/2] \times [(K_p/\cos^2(\varphi)) - 1]$$

where: K_p =factor proposed by Terzaghi, approximable by the following polynomial:

$$K_p = A_0 + A_1 \times \varphi + A_2 \times \varphi^2 + A_3 \times \varphi^3 + A_4 \times \varphi^4;$$

where:

A_0, A_1, A_2, A_3, A_4 =polynomial factors.

(Taking in account that Terzaghi himself advised to use the N_y factor by Meyerhof [see next paragraph]);

c = soil effective cohesion;

y_1 =unit weight above the depth of embedment;

y_2 =unit weight below the depth of embedment;

B =width of the foundation (narrowest side);

D =depth of embedment;

s_c, s_y =shape factors given by:

$s_c = 1.0$ for strip foundation;

$s_c = 1.3$ for square foundation;

$s_y = 1.0$ for strip foundation;

$s_y = 0.8$ for square foundation.

The Terzaghi formula generally gives overestimated values of the bearing capacity, except in case of overconsolidated soils; it has to be used only in case of very shallow foundations, where $D < B$.

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1.2.2 Formula di Meyerhof (1951).

It derives from the Terzaghi formula, to which two new sets of factors are added associated to the depth of embedment and to the inclined loads. Besides a shape factor s_q is also introduced:

$$Q = c \times N_c \times s_c \times d_c \times i_c + s_q \times \gamma_1 \times D \times N_q \times d_q \times i_q + 0.5 \times \gamma_2 \times B \times N_y \times s_y \times d_y \times i_y;$$

where: N_c, N_q, N_y =adimensional bearing capacity factors, given by:

$$N_q = \exp[\pi \times \operatorname{tg}(\varphi)] \times \operatorname{tg}^2(45 + \varphi/2);$$

$$N_c = (N_q - 1) \times \operatorname{cotg}(\varphi);$$

$$N_y = (N_q - 1) \times \operatorname{tg}(1.4 \times \varphi);$$

s_c, s_q, s_y =shape factors, given by:

$$s_c = 1 + 0.2 \times K_p \times B/L;$$

where $K_p = \operatorname{tg}^2(45 + \varphi/2) \times e$ L =length of the foundation;

$$s_q = s_y = 1 + 0.1 \times K_p \times B/L \text{ for } \varphi > 0;$$

$$s_q = s_y = 1 \text{ per for } \varphi = 0;$$

d_c, d_q, d_y =depth factors, given by:

$$d_c = 1 + 0.2 \times \operatorname{sqr}(K_p) \times D/B;$$

$$d_q = d_y = 1 + 0.1 \times \operatorname{sqr}(K_p) \times D/B \text{ for } \varphi > 0;$$

$$d_q = d_y = 1 \text{ for } \varphi = 0;$$

i_c, i_q, i_y =inclined load factors, given by:

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$$i_c = i_q = (1 - I^\circ/90);$$

where I° =inclination of the load in respect to the vertical direction;

$$i_y = (1 - I^\circ/\varphi^\circ)^2 \text{ for } \varphi > 0;$$

$$i_y = 0 \text{ for } \varphi = 0.$$

The Meyerhof formula can be used for any kind of soil and for depth of embedment up to 4 m. Cannot be used in case of foundation on slope, with tilted base or where is $D > B$.

1.2.3 Brinch Hansen (1970).

It derives from the Meyerhof formula, to which two new sets of factors are added associated to foundations on slope and with tilted base. Shape and depth factors are defined. It has the following expression:

$$Q = c \times N_c \times s_c \times d_c \times i_c \times b_c \times g_c + s_q \times y_1 \times D \times N_q \times d_q \times i_q \times b_q \times g_q + 0.5 \times y_2 \times B \times N_y \times s_y \times d_y \times i_y \times b_y \times g_y \text{ (for } \varphi > 0);$$

$$Q = 5.14 \times C_u \times (1 + s_c + d_c - i_c - b_c - g_c) + y_1 \times D \text{ (for } \varphi = 0);$$

where: N_c, N_q, N_y =adimensional bearing capacity factors, given by, dwhere N_c and N_q have the same form than in the Meyerhof formula, whereas the N_y factor is given by:

$$N_y = 1.5 \times (N_q - 1) \times \tan(\varphi);$$

s_c, s_q, s_y =shape factors, given by:
in case of inclined loads:

$$s_c = 0.2 \times (1 - i_c) \times B/L \text{ for } \varphi = 0;$$

$$s_c = 1 + (N_q/N_c) \times (B/L) \text{ for } \varphi > 0;$$

$$s_q = 1 + (B \times i_q/L) \times \tan(\varphi);$$

$$s_y = 1 - 0.4 \times (B \times i_y/L);$$

i_c, i_q, i_y =inclined load factors;

in case of vertical loads only:

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$$\begin{aligned}sc &= 0.2 \times B/L \text{ for } \varphi=0; \\sc &= 1 + (N_q/N_c) \times (B/L) \text{ for } \varphi>0; \\sq &= 1 + (B/L) \times \text{tg}(\varphi); \\sy &= 1 - 0.4 \times (B/L); \end{aligned}$$

dc,dq,dy=depth factors, given by:

$$\begin{aligned}dc &= 0.4 \times k \text{ for } \varphi=0; \\&\text{where } k=D/B \text{ for } D/B \leq 1 \text{ and } k=\text{atang}(D/B) \text{ for } D/B > 1 \\dc &= 1 + 0.4 \times k; \\dq &= 1 + 2 \times \text{tg}(\varphi) \times [1 - \text{sen}(\varphi)]^2 \times k; \\dy &= 1. \end{aligned}$$

ic,iq,iy=inclined load factors, given by:

$$\begin{aligned}ic &= 0.5 - 0.5 \times \text{sqr}[1 - H/(A \times c)] \text{ for } \varphi=0; \\ic &= iq - (1 - iq)/(N_q - 1) \text{ for } \varphi>0; \\iq &= [1 - 0.5 \times H / (V + A \times c \times \text{cotg}(\varphi))]^5; \\iy &= [1 - 0.7 \times H / (V + A \times c \times \text{cotg}(\varphi))]^5 \text{ for } b^\circ=0; \\iy &= [1 - (0.7 - b^\circ/450) \times H / (V + A \times c \times \text{cotg}(\varphi))]^5 \text{ for } b^\circ>0; \\&\text{where } H=\text{horizontal component of the load}; \\&V=\text{vertical component of the load}; \\&b^\circ=\text{Tilt of the base in respect to the horizontal plane.}; \\&A=\text{effective foundation area}; \end{aligned}$$

bc,bq,by=tilted base factors, given by:

$$\begin{aligned}bc &= b^\circ/147 \text{ for } \varphi=0; \\bc &= 1 - b^\circ/147 \text{ for } \varphi>0; \\bq &= \exp[-2 \times b(\text{rad}) \times \text{tg}(\varphi)]; \\by &= \exp[-2.7 \times b(\text{rad}) \times \text{tg}(\varphi)]; \end{aligned}$$

gc,gq,gy=slope factors, give by:

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$$\begin{aligned}gc &= p^\circ/147 \text{ for } \varphi=0; \\gc &= 1 - p^\circ/147 \text{ for } \varphi>0; \\gq &= gy = (1 - 0.5 \times \text{tg } p^\circ)^5.\end{aligned}$$

1.2.4 Vesic (1973).

It has the following expression:

$$Q = c \times Nc \times sc \times dc \times ic \times bc \times gc + sq \times y1 \times D \times Nq \times dq \times iq \times bq \times gq + 0.5 \times y2 \times B \times Ny \times sy \times dy \times iy \times by \times gy \text{ (for } \varphi>0\text{);}$$

$$Q = 5.14 \times Cu \times (1 + sc + dc - ic - bc - gc) + y1 \times D \text{ (for } \varphi=0\text{);}$$

where: Nc, Nq, Ny =adimensional bearing capacity factors, given by, d where Nc and Nq have the same form than in the Meyerhof formula, whereas the Ny factor is given by:

$$Ny = 2 \times (Nq + 1) \times \text{tg}(\varphi);$$

sc, sq, sy =shape factors equal to the Brinch Hansen formula ones;

dc, dq, dy =depth factors equal to the Brinch Hansen formula ones;

ic, iq, iy =inclined load factors, given by:

$$\begin{aligned}ic &= 1 - m \times H / (A \times c \times Nc) \text{ for } \varphi=0; \\ \text{where } m &= (2 + B/L)/(1 + B/L) \text{ for } H \text{ parallel to } B; \\ m &= (2 + L/B)/(1 + L/B) \text{ for } H \text{ parallel to } L; \\ ic &= iq - (1 - iq)/(Nq - 1) \text{ for } \varphi>0; \\ iq &= [1 - H / (V + A \times c \times \text{cotg}(\varphi))]^m; \\ iy &= [1 - H / (V + A \times c \times \text{cotg}(\varphi))]^{(m+1)};\end{aligned}$$

bc, bq, by =tilted base factors, given by:

$$bc = b^\circ/147 \text{ for } \varphi=0;$$

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$$bc = 1 - b^\circ/147 \text{ for } \varphi > 0;$$
$$bq = by = (1 - b \times \text{tg}(\varphi))^2;$$

gc, gq, gy = slope factors, given by:

$$gc = p^\circ/147 \text{ for } \varphi = 0;$$
$$gc = 1 - p^\circ/147 \text{ for } \varphi > 0;$$
$$gq = gy = (1 - \text{tg } p^\circ)^2.$$

1.2.5 Modified Brinch Hansen formula.

It is a variant of the Brinch Hansen formula, where factors N_y e s_q are defined as follows:

$$N_y = 2 \times (N_q - 1) \times \text{tg}(\varphi);$$
$$s_q = 1 + (B/L) \text{sen}(\varphi).$$

1.2.6 Froelich (1935).

In respect to the formulas seen in the previous paragraphs, this formula does not allow to assess the ultimate load, but the critical one. Critical load is the load that, when overtaken, causes not negligible strain inside the soil layer beneath the foundation without reaching the failure. It is particularly applicable in case of local shear failure. The formula has the following expression:

$$Q_{crit} = N_{crit} \times [y_1 \times D + C \times \text{cotg}(\varphi)] \text{ for } \varphi > 0;$$

where $N_{crit} = \pi / [\text{cotg}(\varphi) - (\pi/2 - \varphi)]$ (φ in radians);
D = depth of embedment
 $Q_{crit} = \pi \times c$ for $\varphi = 0$.

1.2.7 Foundation with eccentric load.

In case of structure which transmits moments to the foundation, vertical load is not centered anymore. If V is the vertical load applied to the foundation and M_l and M_b are the moments acting, respectively, along the B and the L sides, the eccentricity is given by:

$$\begin{aligned}e_b &= M_b/V; \\e_l &= M_l/V;\end{aligned}$$

where e_b = eccentricity along B ;
 e_l = eccentricity along L .

The assessment of the bearing capacity will be executed, using effective sizes given as follows:

$$\begin{aligned}B' &= B - 2 \times e_b; \\L' &= L - 2 \times e_l.\end{aligned}$$

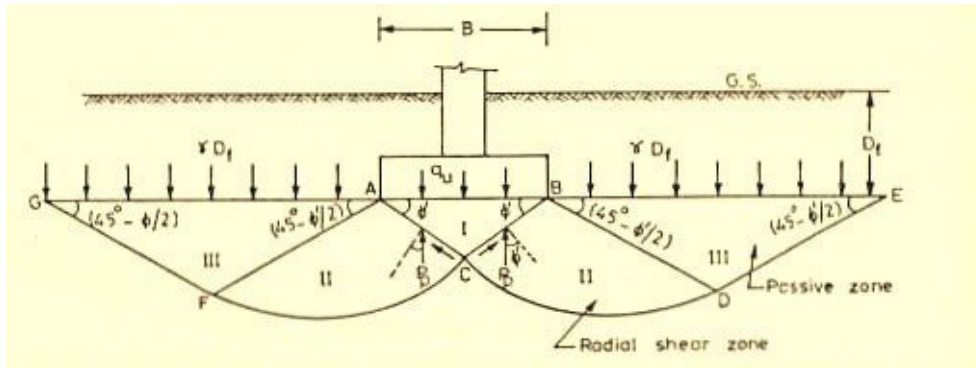
1.2.9 Calculation of the bearing capacity in case of multilayered soils

The depth below the foundation to take in account to calculate the bearing capacity can be estimated after Meyerhof (1953):

$$H = 0.5 \times B \times \text{tg}(45 + \varphi/2);$$

From a practical point of view, H is the thickness of the soil wedge bound to the foundation (zone I).

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If multiple layers lie inside this thickness, the assess of the bearing capacity become more complex.

They can generally distinguish three different cases.

- a) multilayered soil composed by cohesive layers only ($\phi=0$);
- b) multilayered soil composed by granular layers only ($\phi>0$);
- c) multilayered soil composed both by cohesive and granular layers.

a) Meyerhof and Brown (1969) proposed to adopt the following procedure, in case of two-layer soil:

- 1) the ratio between the cohesion of the first and of the second layer, below the foundation, is calculated:

$$R_c = c_1/c_2;$$

- 2) if R_c is less than , a new value of N_c is calculated as follows:

$$N_c = (1.5 \times d/B) + 5.14 \times R_c \quad (N_c \leq 5.14);$$

where: d =thickness of the layer 1;

- 3) If R_c is more than or equal to 1, two different partial factors are calculated:

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$$Nc1 = 4.14 + (0.5 \times B/d);$$

$$Nc2 = 4.14 + (1.1 \times B/d);$$

Nc is given, averaging the two factors:

$$Nc = 2 \times [Nc1 \times Nc2 / (Nc1 + Nc2)].$$

4) The calculated Nc factor is inserted in one of the formulas previously seen (Terzaghi, Meyerhof, etc.) and the bearing capacity Q is calculated.

5) Q is compared with the punching load of the first layer given by:

$$Q_{pz} = 4 \times c1 + y1 \times D;$$

The chosen bearing capacity is the minimum between the two values.

b) Purushothamaray et alii (1974), in case of two layers, proposed the following solution:

1) an average value of ϕ is calculated:

$$\phi' = [d \times \phi1 + (H - d) \times \phi2] / H;$$

where: $\phi1$ and $\phi2$ = angles of internal friction of the layers 1 and 2;

2) an average value of c, if present, is calculated:

$$c' = [d \times c1 + (H - d) \times c2] / H;$$

where: $c1$ and $c2$ = effective cohesions of the layers 1 and 2;

3) the new values of c' and ϕ' are used to calculate the bearing capacity;

4) In case the first layer has poor mechanical characteristics, the punching load has to be calculated, and this value is compared with the bearing capacity of the point 3), then adopting the minimum value.

This procedure can be easily extended to the case of more than two soil layers.

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c) Bowles (1974) in case of two layers, proposed the following solution:

- 1) the bearing capacity Q_1 of the first layer underneath the foundation is calculated through the methods seen in the previous paragraphs (Terzaghi, Meyerhof, etc.);
- 2) the bearing capacity Q_2 of the second layer underneath the foundation is calculated, using c' e φ of the second layer and imposing a value of $\gamma_1 \times D$ given by the product between the unit weight of the first layer and its thickness;
- 3) finally Q' is calculated through the expression:

$$Q' = Q_2 + [p \times P_v \times K \times \text{tg}(\varphi)/A] + (p \times d \times c/A);$$

where: A =foundation area= $B \times L$;

p =foundation perimeter= $2 \times B + 2 \times L$;

d =thickness of the first layer;

P =lithostatic effective pressure calculated from the foundation to the top of second layer;

$K = \text{tg}(45 + \varphi/2)^2$;

- 4) Q' is compared with Q_1 and the minimum value is adopted as bearing capacity.

This procedure can be easily extended to the case of more than two soil layers.

1.2.10 Terzaghi correction of the bearing capacity.

All the calculation methods previously seen are based on the hypothesis the soil underneath the foundation has a behaviour that can be described by the Mohr-Coulomb law:

$$T = c + P_{ef} \times \text{tg}(\varphi);$$

where: T =soil shear strength;

P_{ef} =lithostatic effective pressure.

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Experimental data confirm that this law is generally valid in the range of Q 0-0.45 MPa. Beyond 0.45 MPa the relationship between stress and shear strength is not linear anymore, but it turns in a more complex form. In this case, adopting these formulas the calculated value of Q usually results overestimated.

Terzaghi (1943) suggested to use a correction to apply to the parameters cohesion and angle of shear resistance of the foundation soil, when $Q > 0.45$ MPa, to take in account the not linearity of the stress-shear resistance relationship. In pratica ha suggerito di utilizzare nel calcolo valori ridotti di φ e c , calcolati come segue:

$$c' = (2/3) \times c;$$
$$\varphi' = \text{atang}[(2/3) \times \varphi].$$

The same correction can be applied in case of foundation soil with local shear failure. From a practical point of view, to distinguish between soils undergoing local or general shear failure, one can adopt the following criterion:

a) local shear failure: it is probable in case of foundation soil with relative density ($Dr\%$) less than 20 and/or with a cohesion (c) less than 0.025 MPa; in this case one suggests to proceed to the calculation of Q , adopting the corrected values of c and φ .

$$c' = (2/3) \times c;$$
$$\varphi' = \text{atang}[(2/3) \times \varphi];$$

b) general shear failure: it is probable in case of foundation soil with $Dr\% \geq 70$ and/or with a cohesion higher than or equal to 0.1 MPa; In this case they have to adopt the real values of c and φ , without corrections;

c) intermediate shear failure: it is probable in case of foundation soil with $Dr\% \geq 20$ and < 70 and/or with $c \geq 0.025$ MPa and $c < 0.1$ MPa: In this case one can proceed interpolating between the real values and the corrected values of c and φ .

1.2.11 Foundation on rock mass.

To assess the bearing capacity of a foundation lying on a rock mass, one can use the criterion suggested by Stagg e Zienkiewicz (1968). According to the Authors, the bearing capacity can be calculated using the previously seen methods, adopting the following bearing capacity factors: :

$$\begin{aligned}N_q &= \text{tg}^6(45 + \varphi/2); \\N_c &= 5 \times \text{tg}^4(45 + \varphi/2); \\N_y &= N_q + 1.\end{aligned}$$

Then the resulting value of Q has to be corrected as a function of R.Q.D. (Rock Quality Designation):

$$Q' = Q \times (\text{RQD}\%/100)^2.$$

1.2.12 Bearing capacity in seismic condition.

Cinematic effects on the foundation soil.

In presence of tangential seismic forces, one has to take in account the cinematic effects on the foundation soil, which take to a reduction of the bearing capacity Q.

Vesic and Sano & Okamoto They proposed to quantify the effect, reducing the shear resistance parameters adopted in the bearing capacity calculation.

a)Vesic.

This Author simply suggested to reduce the angle of shear resistance of the foundation soil up to 2 °, whatever is the seismic acceleration.

b)Sano.

Sano proposed to reduce φ as a function of the maximum horizontal seismic acceleration at the depth of embedment of the foundation.

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$$\Delta\varphi = \operatorname{arctg}\left(\frac{a_g}{\sqrt{2}}\right)$$

where a_g is the seismic acceleration.

As an alternative, some Authors propose to act on the bearing capacity factors N_q , N_c e N_γ . Paolucci and Pecker suggest the following corrective factors:

$$z_q = z_\gamma = \left(1 - \frac{k_{hk}}{\operatorname{tg}\varphi}\right)^{0.35}$$
$$z_c = 1 - 0.32k_{hk}$$

where k_{hk} is the horizontal seismic coefficient referred to the depth of embedment of the foundation. The corrected bearing capacity factors are given as follows:

$$N_q' = z_q N_q$$
$$N_\gamma' = z_\gamma N_\gamma$$
$$N_c' = z_c N_c.$$

One can frequently impose $z_q = z_c = 1$.

Inclination of resultant load due to the horizontal seismic force.

The horizontal component of the seismic force leads to an inclined resultant of the load burdening on the foundation. The inclination of resultant load to adopt in the calculation of the bearing capacity, in case of a pre-seismic vertical load only, that is in absence of static horizontal load, can be assess, in a cautelative way, through the following relationship:

$$\theta = \operatorname{arctg}(a_g)$$

where:

a_g = maximum horizontal seismic acceleration at the depth of embedment;

A more correct procedure to calculate the load inclination is that which passing through the assessment of the structure design spectrum. First the

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fundamental period of resonance of the building T is calculated, then, inside the horizontal design spectrum, in correspondence of T , the horizontal seismic coefficient of the structure k_{hi} is read. The inclination of the load owing to the horizontal seismic force given by:

$$\theta = \arctg(k_{hi})$$

Eccentricity of the vertical component of the load.

It has finally to be considered the eccentricity of the vertical load owing to the seismic moment applied on the foundation by the seism along the sides B and L . The eccentricity is given by:

$$e = \frac{M}{N}$$

where M is the seismic moment and N is the vertical component of the load applied on the foundation.

1.3 Sliding resistance of the foundation

When the shallow foundation undergoes horizontal forces, e.g. owing to a seism, its sliding resistance has to be checked.

It has generally to be satisfied the following disequation:

$$H \leq S + E$$

where H is the external horizontal force applied to the foundation, S is the shear resistance along the base and E is the passive force, contrasting H . E is usually neglected, for the strain needed to mobilize it is often too large to be tolerated by the structure.

To determine S , two cases are recognized.

1) Drained condition ($\varphi > 0$):

$$S = V \operatorname{tg} \delta$$

where V is the resultant of the external vertical loads acting on the foundation and δ is the soil-foundation angle of internal friction; δ can be gotten by the following table:

Type	δ
Cast-in-place concrete foundation	$\delta = \varphi$
Concrete precast foundation	$\delta = 2/3 \varphi$

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The parameter φ is the angle of shear strength of the soil layer lying underneath the foundation. The effective cohesion, if present, may be overlooked.

In case of horizontal load owing to seismic force only, the force acting on the foundation is given by:

$$H = V k_{hi}$$

where k_{hi} is the horizontal seismic coefficient of the structure. In granular soils the safety factor for the sliding can be simply assess as follows:

$$F_s = \frac{S}{H} = \frac{tg\delta}{k_{hi}}$$

2) Undrained condition ($\varphi=0$):

$$S = A c_u$$

where c_u is the undrained cohesion of the soil layer underneath the foundation and A is the effective area of the foundation base given by:

$$A = BL \cos \omega$$

with ω = tilting of the base compared to the horizontal plane.

1.4 Modulus of subgrade reaction

It is termed contact pressure the pressure for unit of area that the foundation loads on the underlying soil. The modulus of subgrade reaction is termed the relationship between the contact pressure and the corresponding strain of the underlying soil layer, in a Winkler soil model, that is where a lateral spread of the load is missing:

$$k = Q/s.$$

In case of rigid foundation, the modulus of subgrade reaction can be imposed constant. When the foundation is flexible this assumption is not valid. In this case a variable distribution of k is usually considered, with k increasing as a function of the distance from the foundation centre (pseudo-coupled method), bordering two or more concentric strips. To the most internal strip is assigned a width and a length equal to the half of the total

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width and length of the foundation and a value of k equal to the half of the value imposed to the most external area.

The modulus of subgrade reaction can be assess through the Vesic formula (1961):

$$k \text{ (kg/cm}^2\text{)} = (1/B) \times 0.65 \times [(Et \times B^4)/(Ef \times If)]^{(1/12)} \times Et/(1 - p^2);$$

where: Et (kg/cm²)= strain modulus of the soil below the foundation;

Ef (kg/cm²)= elastic modulus of the foundation;

If (cm⁴)= moment of inertia of the foundation;

B (cm)=minor side of the foundation;

p=Poisson's ratio.

As the product $0.65 \times [(Et \times B^4)/(Ef \times If)]^{(1/12)}$ has generally a value close to 1, the formula may be simplified as follows:

$$k \text{ (kg/cm}^2\text{)} = (1/B) \times Et/(1 - p^2).$$

1.5 Stress diffusion beneath the foundation due to the foundation load.

1.5.1 Introduction.

The loading of the foundation leads to a variation of the stress condition in the underlying soil layers. Load tends to spread beneath the foundation, up to a depth approximately equal to 1-4 x B (B=minor side of the foundation). Assessing the diffusion of the load in the soil layers is essential to estimate the foundation settlement.

1.5.2 Newmark method through the Boussinesq equations.

It is based on the assumption that the foundation soil can be considered a semi-infinite, homogeneous, isotropic, weightless half-space. It derives from the integration on a rectangular or square area BxL of the Boussinesq equations.

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From a practical point of view, the increasing of the effective pressure, owing to the shallow load, at the depth z below the foundation, along the vertical line passing through a vertex of the area $B \times L$, is given by:

$$p_z = [Q/(4 \times \pi)] \times (m_1 + m_2);$$

where: $m_1 = [2 \times M \times N \times \sqrt{V} \times (V + 1)] / [(V + V_1) \times V]$;

$m_2 = \text{atang}[(2 \times M \times N \times \sqrt{V}) / (V_1 - V)]$;

where $M = B/z$;

$N = L/z$;

$V = M^2 + N^2 + 1$;

$V_1 = (M \times N)^2$

To assess the load diffusion along more verticals, the total area $B \times L$ has to be divided in smaller areas, summing then the contribution of the single sub-areas.

The Newmark method usually gives overestimated values of the stress inside the soil mass and, consequently, of the settlement too.

1.5.3 Newmark method through the Westergaard equations.

The soil model by Westergaard takes in account the variability of the mechanical behaviour of the soil layers through the Poisson's ratio parameter. Then it may be adopted when the underground is composed by a multilayered soil.

The increasing of the effective pressure, owing to the shallow load, at the depth z below the foundation, along the vertical line passing through a vertex of the area $B \times L$, is given by:

$$pz = [Q/(2 \times \pi \times z^2)] \times \tan^{-1} \{(M \times N) / [a^{1/2} (M^2 + N^2 + a)^{1/2}]\}$$

where:

$$M = B/z, N=L/z;$$

$$a = (1-2m)/(2-2m) \text{ con } m=\text{Poisson 's ratio.}$$

To assess the load diffusion along more verticals, the total area $B \times L$ has to be divided in smaller areas, summing then the contribution of the single sub-areas.

1.6 Assessment of the foundation settlement.

1.6.1 Introduction.

Though the foundation load does not overtake the bearing capacity, the strains owing to the stress diffusion inside the soil mass might lead to settlement intolerable by the structure.

Settlement is due to elastic and plastic strains of the soil layers and, in case of impervious soil (silt and clay), to the slow outflowing of water from the pores (consolidation).

As the geotechnical behaviour varies from a point to another, as well as the load conditions, settlement may locally assume different values.

Settlement measured or calculated in a specific point is termed total settlement, the difference between total settlements in two or more different points is termed differential settlement.

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The total settlement is given by three components:

$$S_{tot} = S_{imm} + S_{con} + S_{sec};$$

where:

S_{imm} =immediate settlement, Due to the initial strain, with no volume variation, of the loaded soil; it is usually prevalent in granular soils;

S_{con} =consolidation settlement, Due to the gradual outflowing of water included in the soil pores; it is prevalent in soil with low permeability (silt and clay);

S_{sec} =secondary settlement, Due to the viscous strain of the solid framework of the soil; it is usually neglectable compared to the other settlements.

Owing to the different behaviour of the granular and cohesive soils, the two cases have to be separately examined.

1.6.2 Settlement in granular soils.

1.6.2.1 Theory of elasticity.

The theory of elasticity assumes the foundation soil has a perfectly elastic behaviour. The expression is the following:

$$S = DH \times Q_z / E_d;$$

where: DH =layer thickness;

Q_z =Stress increase due to the the shallow load calculated at the depth corresponding to half layer.

E_d =strain modulus of the layer.

This method often gives an overestimated result. The calculate settlement correspond to the immediate component only, as the secondary settlement is considered neglectable. The calculated value is valid for flexible foundations only. In case of rigid foundations, the result has to be corrected,

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applying a factor usually sets equal to 0.93. Besides this method is applicable only when the following condition is satisfied:

$$DH < B;$$

with B=minor side of the foundation.

1.6.2.2 Schmertmann(1978).

It may be used to calculate the immediate and secondary settlements, directly using data from cone penetration tests (CPT). It has the following expression:

$$S_{tot} = C1 \times C2 \times Q \times DH \times \Sigma(Iz/E);$$

where:

Q=net load applied to the foundation;

C1=correction factor to take in account the depth of embedment of the foundation:

$$C1 = 1 - 0.5 \times (P/Q);$$

where P=effective lithostatic pressure at the depth of embedment;

C2=correction factor to take in account the secondary settlement:

$$C2 = 1 + 0.21 \times \text{Log} (T/0.1);$$

where:

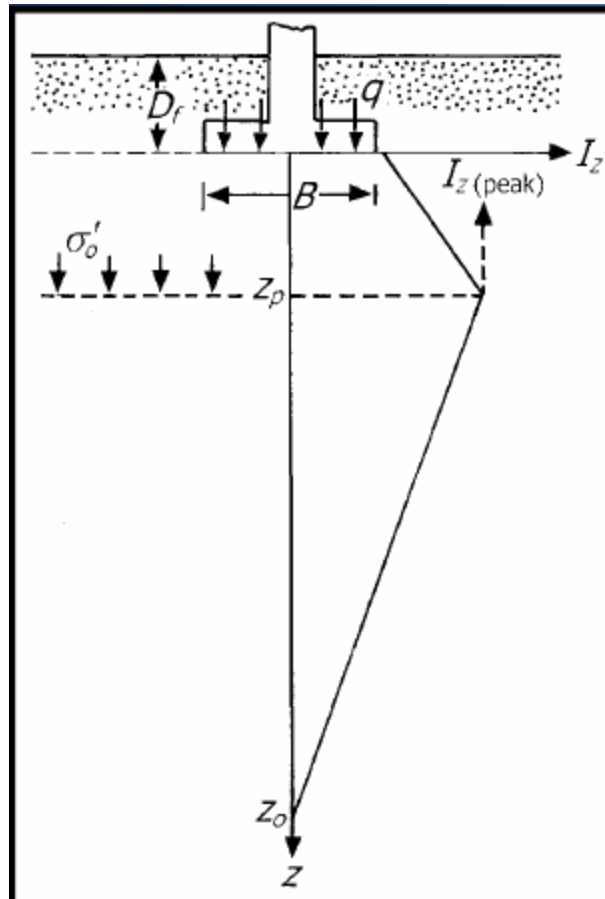
T=calculation time (years);

DH=layer thickness;

E=strain modulus fo the layer;

Iz=factor to take in account distribution of the stress inside the foundation soil owing to the shallow load; it depends on the foundation shape.

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	L/B=1	L/B≥10
Iz a z=0	0.1	0.2
Zp/B	0.5	1.0
Z0/B	2.0	4.0
E	2.5 qc	3.5 qc

Iz starts from a value, corresponding to the case z=0, and increases up to a peak value given by:

$$I_{z(\text{peak})} = 0.5 + 0.1(Q/\sigma_0)^{0.5}$$

At the depth Z0, Iz turns null.

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In case of rectangular foundation, the two former cases ($L/B \geq 10$ and $L/B=1$) are solved and one considers an interpolating value.

One suggests to use the E values proposed by the Author. This method is valid for rigid foundations only.

1.6.2.3 Salgado (2008).

It is a variant of the Schmertmann's method, where I_z assumes the following values:

	I_z
I_z a $z=0$	$0.1+0.0111(L/B) \leq 2$
Z_p/B	$0.5+0.0555(L/B - 1) \leq 1$
Z_0/B	$2.0+0.222(L/B - 1) \leq 4$

1.6.2.4 Lee et al. (2008).

It is a variant of the Schmertmann's method, where I_z assumes the following values:

	I_z
I_z a $z=0$	0.2
Z_p/B	$0.5+0.11(L/B - 1) \leq 1$
Z_0/B	$0.95 \cos \{[(\pi/5)(L/B - 1)] - \pi\} + 3 \leq 4$

The peak value of I_z is given by:

$$I_{z(\text{picco})} = 0.5$$

1.6.2.5 Klein and Sperling (seismic induced settlement).

In loose granular soils, the vibrations induced by the seism might produce an increase of the relative density of foundation layers, that is a decrease of the their volume. Klein and Sperling (2003) suggest a simplified method to assess the consequent settlement based on the following expression:

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$$s(mm) = \frac{e_k - e}{1 - e} H$$

where:

e_k	<p>= $e_{\min} + (e_{\max} - e_{\min})^{0.75a}$ with e_{\min}=minimum void ratio of the soil; e_{\max}=maximum void ratio of the soil; a=horizontal seismic acceleration (g); The decreasing of this parameter as a function of the depth may be taking in account through this formula:</p> $a = 0.65 \frac{a_{\max}}{g} r_d$ <p>where a_{\max} is the maximum horizontal acceleration at the depth of embedment of the foundation and r_d is a correction factor as a function of the depth as follows:: $r_d = 1 - 0.00765z$ for $z \leq 9.15$ m $r_d = 1.174 - 0.0267z$ for $9.15 < z \leq 23$ m $r_d = 0.774 - 0.008z$ for $23 < z \leq 30$ m $r_d = 0.5$ per $z > 30$ m</p>
e	= natural void ratio of the foundation soil.
H	= layer thickness (m).

1.6.3 Settlement in cohesive soils.

1.6.3.1 Theory of elasticity.

It has the same expression seen in the case of granular soils. Instead of the elastic modulus, the edometric modulus has to be used.

1.6.3.2 Edometric method.

It is based on the parameters obtained by consolidation tests; it has the following expression:

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$S_c = DH \times [C_c / (1 + e_0)] \times \text{Log}[(P_f + dp) / P_f]$
(normally consolidated layer);

$S_c = DH \times [C_c / (1 + e_0)] \times \text{Log}[(P_f + dp) / P_f]$
(overconsolidate layer with $dp < P_c$);

$S_c = D_h \times [C_c / (1 + e_0)] \times \text{Log}(P_c / P_f) + DH \times [C_r / (1 + e_0)] \times \text{Log}[(P_f + dp) / P_c]$
(overconsolidated layer with $dp > P_c$);

where: DH=layer thickness;

Cc=compression index;

Cr=recompression index;

Pf=effective stress at half layer;

Pc=overconsolidation stress at half layer;

dp=stress increase due to the shallow load at half layer;

e0=initial void ratio;

In a layered soil the method has to be applied to each layer and the results summed.

To calculate the secondary settlement the following expression is used:

$$S_s = DH \times C_s \times \text{Log}(1 + T);$$

where: Cs=secondary compression index;

T=calculation time (years).

This method is valid in case of one-dimensional consolidation. This condition is verified when:

$$DH < B;$$

with B=minor side of the foundation.

The calculated value is valid for flexible foundations only. In case of rigid foundations, the result has to be corrected, applying a factor usually sets equal to 0.93.

1.6.3.3 Time of consolidation in a cohesive layer.

Consolidation is due to the slow outflowing of water from the soil pores due to applying of the shallow load. Necessary time to attain a specific percentage of the consolidation settlement can be assess by the following expression:

$$t = T \times H^2 / cv;$$

where: T=time ftime factor put in table as a function of the pore pressure distribution in the layer;
H=DH/2, in case of outflow allowed through at both the top and the bottom of the soil layer;
H=DH, in case of outflow allowed through at either the top or the bottom of the soil layer
cv=vertical consolidation coefficient.

In case of a percentage of consolidation equal to 50%, the relationship turns in

$$t = 0.197 \times H^2 / cv.$$

1.6.4 Total and differential settlemens.

High differential settlements might induce damages in a structure. Based on the assumption that high total settlements should produce high differential settlements, Terzaghi and Peck suggested to consider, as maximum tolerable total settlement, a limit value of 2.5 cm in case of granular soil (sand and gravel) and 4 cm in case of cohesive soil (silt and clay). Higher tolerance allowed for cohesive soil depends on the observation that in this kind of material the settlements are mainly due to consolidation and consequently extended in time, allowing the structure to adapt itself to the soil strain.

The angular distortion between two points, whose total settlements are known, is given by:

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$$D_{ang} = (S_2 - S_1) / L_{12};$$

with

D_{ang} =distorsione angolare;

S_2 =maximum settlement in point 2;

S_1 =maximum settlement in point 1;

L_{12} =distance between 1 and 2.

To a first approximation, they are allowed angular distortions less than 1/600 in masonry structures and less than 1/1000 in concrete structures .

1.7 Foundation on expansive soils

In specific climatic condition, usually in arid or semi-arid regions, shallow clayey layers may be undergo swelling when they absorb water. The consequent heave owing to this phenomenon might damage the structure.

An approximately assessment of the dimension of the heave may be obtain, applying some empirical formulas, based on geotechnical parameters of simply determination, as the natural water content and the liquid limit of the expansive layer.

In case of no external loads acting on the expansive layer, the heave S may be calculated by the Johnson and Snethen formula (1979):

$$\text{Log}_{10} S (\%) = 0,0367 w_l - 0,0833 w_n + 0,458$$

or, as an alternative, through the relationship by O'Neill and Ghazzaly (1977):

$$S (\%) = 2,27 + 0,131 w_l - 0,27 w_n$$

where w_l is the liquid limit of the layer and w_n the natural water content.

In case of external loads acting on the expansive layer, the calculated heave has to be corrected to take in account the effect of the stress induced in the soil:

$$S' (\%) = S(1 - 0,0735 q_v^{1/2})$$

where q_v is the load, given in kPa, applied to the top of the soil layer.

Heave is express as a percentage of the layer thickness. Then, to get the total heave, one has to apply the following relationship:

$$s (\text{mm}) = (S'/100) D$$

where D is the thickness of the expansive soil layer.