PROGRAM GEO – QSB-rock ver.2 for Windows

CONTENTS

Theoretical fundamentals

1 Bearing capacity of shallow foundations.

1.1 Introduction.

By the term foundation one refers to the structure fitted to transmit the load of the building and other surcharges acting on it to the underground. The global load has not to overtake the maximum shear strenght of the soil layers. If this would happen, the foundation will undergo a sudden shear failure associated to wide settlements, not tolerable by the building. The maximum theoretical load that a foundation can support immediately before the failure is termed bearing capacity.
Foundation is defined 'shallow' if the following relation is satisfied:

$$D < 4 \times B;$$

where D is the depth of embedment below the ground surface and B is the width of the foundation (B less than or equal to L, length of the foundation). Otherwise the foundation is defined a deep foundation.

1.2 Bearing capacity through analytical methods

Bearing capacity of a foundation on rock depends on several factors:
1. type of rock;
2. joint orientation;
3. joint spacing;
4. joint condition(close or open, weathered or unweathered).

On the base of what suggested by Sowers(1979) and Kulhawy and Goodman(1980), they can be distinguished three different cases.

Intact rock mass.

As regard to the bearing capacity of a foundation on rock, the rock mass can be considered intact, that is not fractured, when the joint spacing is wider than the sizes of the foundation. In this case the bearing capacity depends on the mechanical strength of the rock mass only.

It can consider two situations.

1. Ductile rock: the shear failure is of general sort, with a well-defined wedge-shaped failure surface, which reaches the ground. Calculation can be brought back to the classic formulas of the bearing capacity in soils, employing cohesion and angle of shear strength of the rock mass:

$$q_{\lim} = cNc + \gamma_1 DNq + \frac{1}{2} BN\gamma_2$$

where:

c = cohesion;

γ_1 = unit weight of the rock mass above the depth of embedment;

γ_2 = unit weight of the rock mass beneath the depth of embedment;

D = depth of embedment of the foundation;

B = width of the foundation;

Nc = bearing capacity factor = $2\sqrt{N\phi}\left(N\phi + 1\right)$;

Nq = bearing capacity factor = $N\phi^2$;

Nγ = bearing capacity factor = $\sqrt{N\phi}\left(N\phi^2 - 1\right)$;

Nϕ = $\tan^2\left(45 + \frac{\varphi}{2}\right)$;

φ = angle of shear strength.

2. Brittle rock: in this case a local shear failure can be observed, displaying itself through an initial rock fracturing close to the foundation borders, which propagates beneath the foundation with complex shear failure surfaces. They do not reach the ground surface, but ended inside the rock mass. Calculation of the bearing capacity can be executed through the

classic formulas used in case of soil layers, neglecting the member concerning the depth of embedment of the foundation.

$$q_{\lim} = cNc + \frac{1}{2} BN\gamma_2 .$$

Fractured rock mass.

in case the rock mass be crossed by one or more joint sets with a spacing less or equal to the foundation sizes, the bearing capacity can be influenced by the shear strength of the rock joints, always lower than the rock-mass one. Four cases can be considered.

1. Opened rock joints(>5 mm) with subvertical inclination (>70°): In this case failure occurs when the unconfined compressive strength of the single rock columns, isolated by the joints, is exceeded. The bearing capacity can be calculated by the following formula:

$$q_{\lim} = 2c \tan\left(45 + \frac{\varphi}{2}\right).$$

2. Closed rock joints (≤5 mm) with subvertical inclination (>70°): In this situation the bearing capacity depend on the shear strength of the rock joints only. The following relation cna be applied:

$$q_{\lim} = cNc + \gamma_1 DNq + \frac{1}{2} BN\gamma_2$$

   keeping in mind that cohesion and angle of shear strength have to be referred to the rock joints.
3. Closed or opened rock joints with an inclination between 20° and 70°: In this case too the bearing capacity depend on the shear strength of the rock joints only.
4. Closed or opened rock joints with an inclination <20°: calculation can be brought back to the case of intact rock mass.

Intensely-fractured rock mass (GSI<25).

In presence of two or more joint sets with a very strict spacing, the mechanical behavior of the rock mass can be assimilated to that of a granular soil. Practically the cohesion of the rock mass is neglected, using the angle of shear strength only (c=0, φ>0). The following relation can be applied:

$$q_{\lim} = \gamma_1 D N q + \frac{1}{2} B N \gamma_2$$

1.2.1 Terzaghi (1943).

The Terzaghi formula has the following form:

Q = c x Nc x sc + y1 x D x Nq + 0.5 x y2 x B x Ny x sy;

where:
Nc,Nq,Ny = adimensional bearing capacity factors associated, respectively, to the contribute from cohesive layers, from the weight of the soil above the depth of embedment and from granular layers.
Terzaghi suggested the following relationships:

$$Nq = a^{2 /[2 \text{ x } \cos2(45 + \varphi/2)]}$$

where a = exp[(0.75 x π - φ/2) x tg(φ)];

Nc = (Nq -1) x cotg(φ)

$$Ny = [tg(\varphi)/2] \text{ x } [(Kp/\cos^2(\varphi)) - 1]$$

where: Kp=factor proposed by Terzaghi, approximable by the following polynomial:

$$Kp= A0 + A1 \text{ x } \varphi + A2 \text{ x } \varphi^2 + A3 \text{ x } \varphi^3 + A4 \text{ x } \varphi^4;$$

where:

A0,A1,A2,A3,A4=polynomial factors.

(Taking in account that Terzaghi himself advised to use the Ny factor by Meyerhof [see next paragraph]);

c = soil effective cohesion;
y1=unit weight above the depth of embedment;
y2=unit weight below the depth of embedment;
B=width of the foundation (narrowest side);
D=depth of embedment;
sc,sy=shape factors given by:

sc = 1.0 for strip foundation;
sc = 1.3 for square foundation;
sy= 1.0 for strip foundation;
sy=0.8 for square foundation.

The Terzaghi formula generally gives overestimated values of the bearing capacity, except in case of overconsolidated soils; it has to be used only in case of very shallow foundations, where D<B.

1.2.2 Meyerhof (1951).

It derives from the Terzaghi formula, to which two new sets of factors are added associated to the depth of embedment and to the inclined loads. Besides a shape factor sq is also introduced:

$$Q = c \times Nc \times sc \times dc \times ic + sq \times y1 \times D \times Nq \times dq \times iq + 0.5 \times y2 \times B \times Ny \times sy \times dy \times iy;$$

where: Nc,Nq,Ny=adimensional bearing capacity factors, given by:

$$Nq = \exp[\pi \times tg(\varphi)] \times tg^2(45 + \varphi/2);$$

$$Nc = (Nq - 1) \times cotg(\varphi);$$

$$Ny = (Nq - 1) \times tg(1.4 \times \varphi);$$

sc,sq,sy=shape factors, given by:

sc =1 + 0.2 x Kp x B/L;
where Kp=tg^2(45 + φ/2) e L=lenght of the foundation;
sq = sy = 1 + 0.1 x Kp x B/L for φ>0;
sq = sy = 1 per for φ=0;

dc,dq,dy=depth factors, given by:

dc = 1 + 0.2 x sqr(Kp) x D/B;
dq = dy = 1 + 0.1 x sqr(Kp) x D/B for φ>0;
dq = dy =1 for φ=0;

ic,iq,iy=inclined load factors, given by:

ic = iq = (1 - I°/90);
where I°=inclination of the load in respect to the vertical direction;
iy = (1 - I°/φ°)^2 for φ>0;
iy=0 for φ=0.

The Meyerhof formula can be used for any kind of soil and for depth of embedment up to 4 m. Cannot be used in case of foundation on slope, with tilted base or where is D>B.

1.2.3 Brinch Hansen (1970).

It derives from the Meyerhof formula, to which two new sets of factors are added associated to foundations on slope and with tilted base. Shape and depth factors are defined. It has the following expression:

Q = c x Nc x sc x dc x ic x bc x gc + sq x y1 x D x Nq x dq x iq x bq x gq + 0.5 x y2 x B x Ny x sy x dy x iy x by x gy (for φ>0);

Q = 5.14 x Cu x (1 + sc + dc -ic -bc - gc) + y1 x D (for φ=0);

where: Nc,Nq,Ny=adimensional bearing capacity factors, given by, dwhere Nc and Nq have the same form than in the Meyerhof formula, whereas the Ny factor is given by:

Ny = 1.5 x (Nq –1) x tg(φ);

sc,sq,sy=shape factors, given by:
in case of inclined loads:

sc = 0.2 x (1- ic) x B/L for φ=0;
sc = 1 + (Nq/Nc) x (B/L) for φ>0;
sq = 1 + (B x iq/L) x tg(φ);
sy = 1 – 0.4 x (B x iy/L);
ic,iq,iy=inclined load factors;

in case of vertical loads only:
sc = 0.2 x B/L for φ=0;
sc = 1 + (Nq/Nc) x (B/L) for φ>0;
sq = 1 + (B/L) x tg(φ);
sy = 1 – 0.4 x (B/L);

dc,dq,dy=depth factors, given by:

dc = 0.4 x k for φ=0;
where k=D/B for D/B<=1 and k=atang(D/B) for D/B>1
dc = 1 + 0.4 x k;
dq = 1 + 2 x tg(φ) x [1 - sen(φ)]2 x k;
dy = 1.

ic,iq,iy=inclined load factorsi, given by:

ic = 0.5 - 0.5 x sqr[1 - H/(A x c)] for φ=0;
ic = iq - (1 - iq)/(Nq -1) for φ>0;
iq = [1 - 0.5 x H /(V + A x c x cotg(φ))]5;
iy = [1 - 0.7 x H /(V + A x c x cotg(φ))]^5 for b°=0;
iy = [1 - (0.7-b°/450) x H /(V + A x c x cotg(φ))]^5 for b°>0;
where H=horizontal component of the load;
V=vertical component of the load;
b°=Tilt of the base in respect to the horizontal plane.;
A=effective foundation area ;

bc,bq,by=tilted base factors, given by:

bc = b°/147 for φ=0;
bc = 1 - b°/147 for φ>0;
bq = exp[-2 x b(rad) x tg(φ)];
by = exp[-2.7 x b(rad) x tg(φ)];

gc,gq,gy=slope factors, give by:

gc = p°/147 for φ=0;
gc = 1 - p°/147 for φ>0;
gq = gy = (1 - 0.5 x tg p°)5.

1.2.4 Vesic (1973).

It has the following expression:

Q = c x Nc x sc x dc x ic x bc x gc + sq x y1 x D x Nq x dq x iq x bq x gq + 0.5 x y2 x B x Ny x sy x dy x iy x by x gy (for φ>0);

Q = 5.14 x Cu x (1 + sc + dc -ic -bc - gc) + y1 x D (for φ=0);

where: Nc,Nq,Ny=adimensional bearing capacity factors, given by, dwhere Nc and Nq have the same form than in the Meyerhof formula, whereas the Ny factor is given by:

$$Ny = 2 \times (Nq + 1) \times tg(\varphi);$$

sc,sq,sy=shape factors equal to the Brinch Hansen formula ones;

dc,dq,dy=depth factors equal to the Brinch Hansen formula ones;

ic,iq,iy=inclined load factors, given by:

$ic = 1 - m \times H / (A \times c \times Nc)$ for $\varphi=0$;
where $m=(2 + B/L)/(1 + B/L)$ for H parallel to B;
$m=(2 + L/B)/(1 + L/B)$ for H parallel to L;
$ic = iq - (1 - iq)/(Nq - 1)$ for $\varphi>0$;
$iq = [1 - H /(V + A \times c \times cotg(\varphi))]m$;
$iy = [1 - H /(V + A \times c \times cotg(\varphi))]^{(m+1)}$;

bc,bq,by=tilted base factors, given by:

$bc = b°/147$ for $\varphi=0$;
$bc = 1 - b°/147$ for $\varphi>0$;
$bq = by = (1 - b \times tg(\varphi))^2$;

gc,gq,gy=slope factors, given by:

$gc = p°/147$ for $\varphi=0$;
$gc = 1 - p°/147$ for $\varphi>0$;
$gq = gy = (1 - tg\ p°)^2$.

1.2.5 Modified Brinch Hansen formula.

It is a variant of the Brinch Hansen formula, where factors Ny e sq are defined as follows:

Ny = 2 x (Nq -1) x tg(φ);
sq=1 + (B/L)sen(φ).

1.2.6 Instantaneous angle of shearing strength and cohesion of rock mass and discontinuities. Hoek and Brown criterion.

The Coulomb criterion

$$\tau = c + \sigma \tan \varphi;$$

where
c = cohesion;
σ = effective pressure;
φ = angle of shear strength.

cannot be applied to the rock, where the correlation between shear strength and effective pressure is not linear. However it's possible to estimate instantaneous values of cohesion and angle of shear strength, relative to a specific value of effective pressure, through the empirical Hoek and Brown criterion.

The criterion is expressed as

$$\sigma_1 = \sigma_{3'} + \sigma_c \left[m_b \frac{\sigma_3}{\sigma_c} + s \right]^a ;$$

where:

s, a, m_b =	Constants for a specific rock type;
σ_c =	Uniaxial compressive strength of the intact rock;
σ_1 σ_3 =	Major and minor principal stresses.

The s, a and m_b rock constants can be correlated to GSI (Geological Strength Index).
Three cases are distinguished based on the GSI value.

- Undisturbed rock and G.S.I.>25:

$$m = m_i e^{\frac{GSI-100}{28}} \quad s = e^{\frac{GSI-100}{9}}$$
$$a = 0{,}5$$

- Undisturbed rock and G.S.I.≤25:

$$m = m_i e^{\frac{GSI-100}{28}}$$
$$s = 0$$
$$a = 0{,}65 - \frac{GSI}{200}$$

- Disturbed rock any value of G.S.I.

$$m_r = m_i e^{\frac{GSI-100}{14}}$$
$$s_r = e^{\frac{GSI-100}{6}}$$
$$a = 0{,}5$$

where:

m_i= variable depending on the rock mineralogy and petrographic characteristics, derivable from the following table:

Rock type	Class	Group	Texture: Coarse	Medium	Fine	Very fine
SEDIMENTARY	Clastic		Conglomerates* (21 ± 3) Breccias (19 ± 5)	Sandstones 17 ± 4	Siltstones 7 ± 2 Greywackes (18 ± 3)	Claystones 4 ± 2 Shales (6 ± 2) Marls (7 ± 2)
	Non-Clastic	Carbonates	Crystalline Limestone (12 ± 3)	Sparitic Limestones (10 ± 2)	Micritic Limestones (9 ± 2)	Dolomites (9 ± 3)
		Evaporites		Gypsum 8 ± 2	Anhydrite 12 ± 2	
		Organic				Chalk 7 ± 2
METAMORPHIC	Non Foliated		Marble 9 ± 3	Hornfels (19 ± 4) Metasandstone (19 ± 3)	Quartzites 20 ± 3	
	Slightly foliated		Migmatite (29 ± 3)	Amphibolites 26 ± 6		
	Foliated**		Gneiss 28 ± 5	Schists 12 ± 3	Phyllites (7 ± 3)	Slates 7 ± 4
IGNEOUS	Plutonic	Light	Granite 32 ± 3 Diorite 25 ± 5 Granodiorite (29 ± 3)			
		Dark	Gabbro 27 ± 3 Norite 20 ± 5	Dolerite (16 ± 5)		
	Hypabyssal		Porphyries (20 ± 5)		Diabase (15 ± 5)	Peridotite (25 ± 5)
	Volcanic	Lava		Rhyolite (25 ± 5) Andesite 25 ± 5	Dacite (25 ± 3) Basalt (25 ± 5)	Obsidian (19 ± 3)
		Pyroclastic	Agglomerate (19 ± 3)	Breccia (19 ± 5)	Tuff (13 ± 5)	

Instantaneous cohesion (c_i) and angle of shear strength (φ_i) of the rock mass.

The parameters c_i and φ_i can be obtained through an implicit numerical technique. The calculation steps are the following:

- Using the Hoek and Brown criterion, σ_1 is calculated, making σ_3 variable from a value close to 0 to a maximum value approximately equal to 0.25 σ_c. The incremental step of σ_3 ($\Delta\sigma_3$) is given by the ratio $\Delta\sigma_3 = \sigma_c/2^{10}$. To n steps $\Delta\sigma_3$ correspond n couple of σ_1, σ_3 values, through the Hoek and Brown formula, and n sets of values $\delta\sigma_1/\delta\sigma_3$, σ_n', τ, given by the Balmer relations:

$$\sigma_n = \sigma_3 + \frac{\sigma_1 - \sigma_3}{\frac{\delta\sigma_1}{\delta\sigma_3} + 1};$$

$$\tau = (\sigma_n - \sigma_3)\sqrt{\frac{\delta\sigma_1}{\delta\sigma_3}};$$

$$\frac{\delta\sigma_1}{\delta\sigma_3} = 1 + \frac{m_b\sigma_c}{2(\sigma_1 - \sigma_3)} \text{(GSI>25, a=0,5).}$$

$$\frac{\delta\sigma_1}{\delta\sigma_3} = 1 + a m_b{}^a\left(\frac{\sigma_3}{\sigma_c}\right)^{a-1} \text{(GSI≤25, s=0).}$$

By the linear regression formula:

$$\varphi_i' = arc\tan\left[\frac{\sum\sigma_n\tau - \frac{\sum\sigma_n\sum\tau}{n}}{\sum\sigma_n{}^2 - \frac{(\sum\sigma_n)^2}{n}}\right],$$

$$c_i' = \left(\frac{\sum\tau}{n}\right) - \left[\left(\frac{\sum\sigma_n}{n}\right)\tan\varphi_i'\right],$$

- Inside the calculated intervals of σ_n values ($\Delta\sigma_n$), the interval where falls the searched σ_n' value is identified. $\Delta\sigma_n$ is associated to the intervals of cohesion and angle of shear strength (Δc_i' and $\Delta\varphi_i$'), whereby:

$$c_i = \frac{\sigma_{nbc}'}{\Delta\sigma_n}\Delta c_i',$$

$$\varphi_i = \frac{\sigma_{nbc}'}{\Delta\sigma_n}\Delta\varphi_i',$$

Instantaneous cohesion (c_i) and angle of shear strength (φ_i) of the discontinuites.

The shear strength of the discontinuities, expressed as c_i and φ_i values, can be estimated through the relations suggested by Barton.
These the calculation steps:

$$\tau = \sigma_n' \tan\left[\varphi_b + JRCLog_{10}\left(\frac{JCS}{\sigma_n'}\right)\right];$$

$$\frac{\delta\tau}{\delta\sigma_n} = \tan\left[\varphi_b + JRCLog_{10}\left(\frac{JCS}{\sigma_n'}\right)\right] - \frac{\pi JRC}{180\ln 10}\left\{\tan^2\left[\varphi_b + JRCLog_{10}\left(\frac{JCS}{\sigma_n'}\right)\right] + 1\right\}$$

$$;\varphi_i = arc\tan\left(\frac{\delta\tau}{\delta\sigma_n}\right);$$

$$c_i = \tau - \sigma_n \tan\varphi_i .$$

1.2.7 Foundation with eccentric load.

In case of structure which transmits moments to the foundation, vertical load is not centered anymore. If V is the vertical load applied to the foundation and Ml and Mb are the moments acting, respectively, along the B and the L sides, the eccentricity is given by:

$$eb = Mb/V;$$
$$el = Ml/V;$$

where eb = eccentricity along B;
el = eccentricity along L.

The assessment of the bearing capacity will be executed, using effective sizes given as follows:

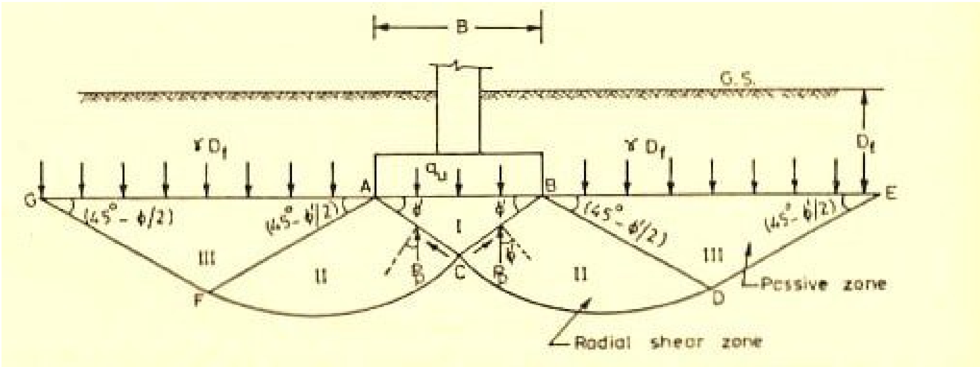
$$B' = B - 2 \times eb;$$
$$L' = L - 2 \times el.$$

1.2.8 Calculation of the bearing capacity in case of multilayered soils

The depth below the foundation to take in account to calculate the bearing capacity can be estimated after Meyerhof (1953):

$$H = 0.5 \times B \times tg(45 + \varphi/2);$$

From a practical point of view, H is the thickness of the soil wedge bound to the foundation (zone I).



If multiple layers lie inside this thickness, the assess of the bearing capacity become more complex.
They can generally distinguish three different cases.

a) multilayered soil composed by cohesive layers only (φ=0);
b) multilayered soil composed by granular layers only (φ>0);
c) multilayered soil composed both by cohesive and granular layers.

a) Meyerhof and Brown (1969) proposed to adopt the following procedure, in case of two-layer soil:

1) the ratio between the cohesion of the first and of the second layer, below the foundation, is calculated:

$$Rc = c1/c2;$$

2) if Rc is less than , a new value of Nc is calculated as follows:

$$Nc = (1.5 \times d/B) + 5.14 \times Rc \ (Nc<=5.14);$$

where: d=thickness of the layer 1;

3) If Rc is more than or equal to 1, two different partial factors are calculated:

$$Nc1 = 4.14 + (0.5 \times B/d);$$
$$Nc2 = 4.14 + (1.1 \times B/d);$$

Nc is given, averaging the two factors:

$$Nc = 2 \times [Nc1 \times Nc2 /(Nc1 + Nc2)].$$

4) The calculated Nc factor is inserted in one of the formulas previously seen (Terzaghi, Meyerhof, etc.) and the bearing capacity Q is calculated.
5) Q is compared with the punching load of the first layer given by:

$$Qpz = 4 \times c1 + y1 \times D;$$

The chosen bearing capacity is the minimum between the two values.

b) Purushothamaray et alii (1974), in case of two layers, proposed the following solution:

1) an average value of φ is calculated:

$$\varphi' = [d \times \varphi 1 + (H - d) \times \varphi 2] / H;$$

where: φ1 and φ2 = angles of internal friction of the layers 1 and 2;
2) an average value of c, if present, is calculated:

$$c' = [d \times c1 + (H - d) \times c2] / H;$$

where: c1 and c2 = effective cohesions of the layers 1 and 2;
3) the new values of c' and φ' are used to calculate the bearing capacity;
4) In case the first layer has poor mechanical characteristics, the punching load has to be calculated, and this value is compared with the bearing capacity of the point 3), then adopting the minimum value.

This procedure can be easily extended to the case of more than two soil layers.

c) Bowles (1974) in case of two layers, proposed the following solution:

1) the bearing capacity Q1 of the first layer underneath the foundation is calculated through the methods seen in the previous paragraphs (Terzaghi, Meyerhof, etc.);
2) the bearing capacity Q2 of the second layer underneath the foundation is calculated, using c' e φ of the second layer and imposing a value of y1xD given by the product between the unit weight of the first layer and its thickness;
3) finally Q' is calculated through the expression:e:

$$Q' = Q2 + [p \times Pv \times K \times tg(\varphi)/A] + (p \times d \times c/A);$$

where: A=foundation area=B x L;
p=foundation perimeter=2 x B + 2 x L;
d=thickness of the first layer;
P=lithostatic effective pressure calculated from the foundation to the top of second layer;
$K=tg(45 + \varphi/2)^2$;
4) Q' is compared with Q1 and the minimum value is adopted as bearing capacity.

This procedure can be easily extended to the case of more than two soil layers.

1.2.9 Bearing capacity in seismic condition.

Cinematic effects on the foundation soil.

In presence of tangential seismic forces, one has to take in account the cinematic effects on the foundation soil, which take to a reduction of the bearing capacity Q.
Vesic and Sano & Okamoto They proposed to quantify the effect, reducing the shear resistance parameters adopted in the bearing capacity calculation.
a)Vesic.

This Author simply suggested to reduce the angle of shear resistance of the foundation soil up to 2 °, whatever is the seismic acceleration.

b)Sano.

Sano proposed to reduce φ as a function of the maximum horizontal seismic acceleration at the depth of embedment of the foundation.

$$\Delta\varphi = arctg\left(\frac{ag}{\sqrt{2}}\right)$$

where a_g is the seismic acceleration.

As an alternative, some Authors propose to act on the bearing capacity factors N_q, N_c e N_γ. Paolucci and Pecker suggest the following corrective factors:

$$z_q = z_\gamma = \left(1 - \frac{k_{hk}}{tg\varphi}\right)^{0.35}$$

$$z_c = 1 - 0.32k_{hk}$$

where k_{hk} is the horizontal seismic coefficient referred to the depth of embedment of the foundation. The corrected bearing capacity factors are given as follows:

$$N_q' = z_q\, N_q$$

$$N_\gamma' = z_\gamma\, N_\gamma$$

$$N_c' = z_c\, N_c.$$

One can frequently impose $z_q = z_c = 1$.

Inclination of resultant load due to the horizontal seismic force.

The horizontal component of the seismic force leads to an inclined resultant of the load burdening on the foundation. The inclination of resultant load to adopt in the calculation of the bearing capacity, in case of a pre-seismic vertical load only, that is in absence of static horizontal load, can be assess, in a cautelative way, through the following relationship:

$$\theta = arctg(a_g)$$

where:
a_g = maximum horizontal seismic acceleration at the depth of embedment;
A more correct procedure to calculate the load inclination is that which passing through the assessment of the structure design spectrum. First the fundamental period of resonance of the building T is calculated, then, inside the horizontal design spectrum, in correspondence of T, the horizontal seismic coefficient of the structure khi is read. The inclination of the load owing to the horizontal seismic force given by:

$$\theta = arctg(k_{hi})$$

Eccentricity of the vertical component of the load.

It has finally to be considered the eccentricity of the vertical load owing to the seismic moment applied on the foundation by the seism along the sides B and L. The eccentricity is given by:

$$e = \frac{M}{N}$$

where M is the seismic moment and N is the vertical component of the load applied on the foundation.

1.3 Sliding resistance of the foundation

When the shallow foundation undergoes horizontal forces, e.g. owing to a seism, its sliding resistance has to be checked.
It has generally to be satisfied the following disequation:

$$H \le S + E$$

where H is the external horizontal force applied to the foundation, S is the shear resistance along the base and E is the passive force, contrasting H. E is usually neglected, for the strain needed to mobilize it is often too large to be tolerated by the structure.
To determine S, two cases are recognized.

1) Drained condition ($\varphi>0$):

$$S = Vtg\delta$$

where V is the resultant of the external vertical loads acting on the foundation and δ is the soil-foundation angle of internal friction; δ can be gotten by the following table:

Type	δ
Cast-in-place concrete foundation	δ=φ
Concrete precast foundation	δ=2/3 φ

The parameter φ is the angle of shear strength of the soil layer lying underneath the foundation. The effective cohesion, if present, may be overlooked.
In case of horizontal load owing to seismic force only, the force acting on the foundation is given by:

$$H = Vk_{hi}$$

where k_{hi} is the horizontal seismic coefficient of the structure. In granular soils the safety factor for the sliding can be simply assess as follows:

$$F_s = \frac{S}{H} = \frac{tg\delta}{k_{hi}}$$

2) Undrained condition (φ=0):

$$S = Ac_u$$

where c_u is the undrained cohesion of the soil layer underneath the foundation and A is the effective area of the foundation base given by:

$$A=BL\cos\omega$$

with ω= tilting of the base compared to the horizontal plane.

1.4 Modulus of subgrade reaction

It is termed contact pressure the pressure for unit of area that the foundation loads on the underlying soil. The modulus of subgrade reaction is termed the relationship between the contact pressure and the corresponding strain of the underlying soil layer, in a Winkler soil model, that is where a lateral spread of the load is missing:

$$k = Q/s.$$

In case of rigid foundation, the modulus of subgrade reaction can be imposed constant. When the foundation is flexible this assumption is not valid. In this case a variable distribution of k is usually considered, with k increasing as a function of the distance from the foundation centre (pseudo-coupled method), bordering two or more concentric strips. To the most internal strip is assigned a width and a length equal to the half of the total width and length of the foundation and a value of k equal to the half of the value imposed to the most external area.
The modulus of subgrade reaction can be assess through the Vesic formula (1961):

$$k\ (kg/cmc) = (1/B) \times 0.65 \times [(Et \times B4)/(Ef \times If)]^{(1/12)} \times Et/(1 - p2);$$

where: Et (kg/cmq)= strain modulus of the soil below the foundation;
Ef (kg/cmq)= elastic modulus of the foundation;
If (cm4)= moment of inertia of the foundation;
B (cm)=minor side of the foundation;
p=Poisson's ratio.

As the product $0.65 \times [(Et \times B4)/(Ef \times If)]^{(1/12)}$ has generally a value close to 1, the formula may be simplified as follows:

$$k\ (kg/cmc) = (1/B) \times Et/(1 - p2).$$

1.5 Stress diffusion beneath the foundation due to the foundation load.

1.5.1 Introduction.

The loading of the foundation leads to a variation of the stress condition in the underlying soil layers. Load tends to spread beneath the foundation, up to a depth approximately equal to 1-4 x B (B=minor side of the foundation). Assessing the diffusion of the load in the soil layers is essential to estimate the foundation settlement.

1.5.2 Newmark method through the Boussinesq equations.

It is based on the assumption that the foundation soil can be considered a semi-infinite, homogeneous, isotropic, weightles half-space. It derives from the integration on a rectangular or square area BxL of the Boussinesq equations.
From a practical point of view, the increasing of the effective pressure, owing to the shallow load, at the depth z below the foundation, along the vertical line passing through a vertex of the area B x L, is given by:

$$pz = [Q/(4 \times \pi)] \times (m1 + m2);$$

where: m1=[2 x M x N x sqr(V) x (V + 1)] / [(V + V1) x V];
m2=atang[(2 x M x N x sqr(V))/(V1 - V)];
where M=B/z;
N=L/z;
$V=M^2 + N2 + 1$;
$V1=(M \times N)^2$

To assess the load diffusion along more verticals, the total area B x L has to be divided in smaller areas, summing then the contribution of the single sub-areas.
The Newmark method usually gives overestimated values of the stress inside the soil mass and, consequently, of the settlement too.

1.5.3 Newmark method through the Westergaard equations.

The soil model by Westergaard takes in account the variability of the mechanical behaviour of the soil layers through the Poisson's ratio parameter. Then it may be adopted when the underground is composed by a multilayered soil.
The increasing of the effective pressure, owing to the shallow load, at the depth z below the foundation, along the vertical line passing through a vertex of the area B x L, is given by:

$$pz = [Q/(2 \times \pi \times z^2)] \times \tan^{-1} \{(M \times N) / [a^{1/2} (M^2 + N^2 + a)^{1/2}]\}$$

where:
M = M=B/z, N=L/z;
a = (1-2m)/(2-2m) con m=Poisson 's ratio.

To assess the load diffusion along more verticals, the total area B x L has to be divided in smaller areas, summing then the contribution of the single sub-areas.

1.6 Assessment of the foundation settlement.

1.6.1 Introduction.

Though the foundation load does not overtake the bearing capacity, the strains owing to the stress diffusion inside the soil mass might lead to settlement intolerable by the structure.
Settlement in rock layers is due to elastic and plastic strains.
As the geotechnical behaviour varies from a point to another, as well as the load conditions, settlement may locally assume different values.
Settlement measured or calculated in a specific point is termed total settlement, the difference between total settlements in two or more different points is termed differential settlement.

Theory of elasticity.

The theory of elasticity assumes the foundation soil has a perfectly elastic behaviour. The expression is the following:

$$S = DH \times Qz / Ed;$$

where: DH=layer thickness;
Qz=Stress increase due to the the shallow load calculated at the depth corresponding to half layer.
Ed=elastic modulus of the layer.

The calculated value is valid for flexible foundations only. In case of rigid foundations, the result has to be corrected, applying a factor usually sets equal to 0.93. Besides this method is applicable only when the following condition is satisfied:

$$DH < B;$$

with B=minor side of the foundation.

1.6.2 Total and differential settlemens.

High differential settlements might induce damages in a structure. Based on the assumption that high total settlements should produce high differential settlements, Terzaghi and Peck suggested to consider, as maximum tolerable total settlement, a limit value of 2.5 cm.
The angular distortion between two points, whose total settlements are known, is given by:

$$Dang= (S2 -S1)/L12;$$

with
Dang=distorsione angolare;
S2=maximum settlement in point 2;
S1=maximum settlement in point 1;
L12=distance between 1 and 2.
To a first approximation, they are allowed angular distortions less than 1/600 in masonry structures and less than 1/1000 in concrete structures .