Theoretical basis.

The concept of hydraulic invariance supposes the making of interventions, inside the area which will be subjected to decrease their superficial permeability because of the designed transformations, with the purpose of keeping unvaried the outflow discharge capacity from the area. This result can be obtained both allowing the infiltration of the rainfall into the subsoil and through the lamination of the superficial outflow.
In this last case the intervention practically consists of building detention basins, with the purpose to stock temporary the water volume due to the rainfall during the meteoric event and then to release it gradually, imposing a discharge capacity not higher than the pre-transformation value.
The types of interventions are mainly the following:

1. detention basins with impervious bottom;
2. green lowered areas;
3. draining trenches;
4. infiltration wells

In case of small water volume it is possible, as an alternative, to oversize the sewer pipes.

Estimating the design rainfall.

Equations which describe the depth of rainfall as a function of their duration are called IDF (Intensity-Duration-Frequency) curves. Equation which links these two variables can generally have the following forms:

a) $h\ (mm) = a\ t^{n}$ (2-variable curve)

b) $h\ (mm) = a\ t\ /\ (t+b)^{c}$ (3-variable curve)

where *a*(mm/h) = rainfall depth when t=1 hour;
b(h)= temporal factor;
n , *c*= scale factors as a function of the rainfall duration.

Putting *c=1-n* and *b=0* formula b) becomes a).
Sizing of detention basins and of green lowered areas, where the volume of infiltration does not exceed 50% of the total rainfall, is usually referred to a return period of 50 years. Sizing of infiltration wells, of draining trenches and of green lowered areas, in this case when the volume of infiltration exceed 50% of the total rainfall, is usually referred to a return period of 100 years in plain areas and of 200 years in hilly areas.

Design of impervious detention basins.

They're basins, usually made in concrete, equipped with a discharge pipe at the bottom. Surface runoff, during the rainfall event, is conducted into the basin and then gradually released through the discharge pipe into a surface waterbody. Sizing of basin is usually performed through the estimate of its minimum volume, taking in account both the inflow water volumes and the outflow ones through the discharge pipe.

Simplified method.

It's based on an empirical formula to get a <u>conservative</u> estimate of the specific volume of the basin:

$$(1)\, W(mc/ha) = w_0 \left(\frac{c_{a2}}{c_{a1}}\right)^{\frac{1}{1-n}} - 15I - w_0 P$$

where:

w_0(mc/ha)	= natural specific volume available inside the transformed area for the lamination of the surface runoff;
c_{a2}	= runoff coefficient after the transformation;
c_{a1}	= runoff coefficient before the transformation;
n	= parameter n of the IDF curve;
15	= specific volume available inside the transformed area for the lamination of the surface runoff after the transformation;

I	= percentage of the transformed area (decimal format);
P	= percentage of the untransformed area (decimal format).

the maximum volume of water which can be stored inside the detention basin is given by the product of W and the transformed area A, expressed in hectares:

$$W_{lam}(mc) = WA$$

the runoff coefficients of the transformed areas are given by a weighted average of the runoff coefficients of the impervious (A_i) and pervious (A_p). areas.

$$(2)\, c_a = \frac{c_{ai} A_i + c_{ap} A_p}{A_i + A_p}$$

Formula 1) is applicable only in case of plain areas.

Rational method.

Applying this method involves that the surface runoff increases as a function of time during the rainfall event until reaching a maximum value at the time t_c.*(time of concentration).* Starting from t_c, all the fluvial basin contributes to the surface runoff and the discharge capacity Q reaches its maximum value. Then Q keeps its value constant till the end of the rainfall.
Time of concentration can be estimated by the formula proposed by Boyd, valid for flat areas of small extension:

$$t_c(ore) = t_0 + t_r$$

where:

$$t_r = \frac{\sqrt{1{,}5A}}{v} \text{ e } t_0 = kA^d$$

and:

A(kmq) = area of the transformed surface;

k = 2,51
d = 0,38
v = 1,00

Rational method implies two hypotheses:

- the rainfall intensity has to be constant;
- emptying of the detention basin has to be at constant discharge (Q_u=cost).

At time t the stored volume inside the basin is given by the following formula:

$$(3)\, W(mc) = c_a Ah + t_c Q_u^{\,2} \frac{t}{c_a Ah} - Q_u t - Q_u t_c$$

where:

c_a	= runoff coefficient;
A	= area of the transformed surface;
h	= rainfall depth given by the selected IDF curve.

The rainfall duration (t_r=critical duration) which generates the maximum stored volume is given by the first derivative of the formula 3). Inserting the estimated tr value into the formula 3), the maximum storage volume is calculated.

Direct rainfall method.

This method involves that the maximum surface runoff will be immediately reached at the beginning of the rainfall event. Direct rainfall method implies two hypotheses:

- the rainfall intensity has to be constant;
- emptying of the detention basin has to be at constant discharge (Q_u=cost).

At time t the stored volume inside the basin is given by the following formula:

$$(5)\, W(mc) = c_a Ah - Q_u t$$

where:

c_a	= runoff coefficient;
A	= area of the transformed surface;
h	= rainfall depth given by the selected IDF curve.

The rainfall duration (t_r=critical duration) which generates the maximum stored volume is given by the first derivative of the formula 5). Inserting the estimated tr value into the formula 5), the maximum storage volume is calculated.

Reservoir method.

This method involves that the basin behaves as a linear reservoir.
Reservoir method implies two hypotheses:

- the rainfall intensity has to be constant;
- emptying of the detention basin has to be at constant discharge (Q_u=cost).

Critical discharge of the basin is given by the following formula:

$$(7)\, Q_c(mc/s) = c_a AaDk^{n-1}$$

in cui:

c_a	= runoff coefficient;
A	= area of the transformed surface;
a	= parameter a of the selected IDF curve;
k	= reservoir constant of the basin.

Variable D is calculable through the following expression:

$$(8)\, D = C^{n-1}\left(1 - e^{-C}\right)$$

where:

$$(9)\, n = \frac{1 + C - e^C}{1 - e^C}$$

The rainfall duration (t_r=critical duration) which generates the maximum stored volume is given by:

$$(10)\, t_r = kF$$

where F is gotten by the resolution of the following formula:

$$(11)\, nF + (1-n)\ln\left(\frac{\frac{m}{D}F^{n-1}}{\frac{m}{D}F^{n-1}-1}\right) - \frac{\frac{D}{m}F^{2-n}}{1-e^{F}} = 0$$

Parameter m is defined by the ratio Q_c/Q_u ($m = Q_c/Q_u$).
Finally the maximum storage volume is given by:

$$(12)\, W(mc) = kQ_c G$$

where G is given by the following equation:

$$(13)\quad G = \frac{F^{n}}{D} - \frac{F^{n-1}}{D}\ln\left(\frac{\frac{m}{D}F^{n-1}}{\frac{m}{D}F^{n-1}-1}\right) - \frac{F}{m} - \frac{1}{m}\ln\left[\left(\frac{mF^{n-1}}{D}-1\right)\left(1-e^{-F}\right)\right]$$

Sizing of the discharge pipe and estimating the area of the basin.

The diameter of the discharge pipe is a function of the maximum hydraulic depth inside the detention basin. It can be calculated by the following formula (Giorgi, 2004):

$$Q = 0,6\pi\left(\frac{D}{2}\right)^2 \sqrt{2gh}$$

where:

Q(mc/s)	= outflow capacity of the discharge pipe;
D(m)	= pipe diameter;
h(m)	= hydraulic depth;
g(m/s^2)	= gravity acceleration = 9,81.

The outflow capacity is known, thus the formula can be used to get:

- ❑ diameter D, fixed h;
- ❑ hydraulic depth h, fixed D.

Known the variable h, the basin area is given by the ratio between the basin volume and the hydraulic depth:

$$A = \frac{W}{h}$$

Sizing of green lowered surfaces.

They're areas, with a permeable bottom, lowered in respect to the contiguous ground. Surface runoff, during the rainfall event, is conducted into the lowered surface and then a part is gradually released through the discharge pipe into a surface waterbody, while the rest is taken away through infiltration on the bottom . Sizing of green lowered surfaces is usually performed through the estimate of its minimum volume, taking in account both the inflow water volumes and the outflow ones through the discharge pipe and infiltration.

Estimating the surface runoff.

Surface runoff can be calculated through the explained methods in the previous paragraphs, particularly using the rational and direct rainfall methods.

Estimating the potential infiltration ratio.

The potential infiltration ratio (f) is the maximum water volume which can be infiltrated into the ground, if such a volume is available. The actual infiltration water volume may be lower if the surface runoff is not sufficient. Anyway it cannot be higher.
The potential infiltration ratio depends on the ground permeability and on the initial saturation ratio. The higher is the permeability, the higher will be the infiltration. The higher is the saturation ratio, the lower will be the infiltration.
Green & Ampt's method is commonly used to estimate the potential infiltration ratio. This procedure involves that the saturation front moves itself downward as a function of the time, dividing distinctly the saturated ground volume, with a water contents equal to the soil porosity (η), by the deeper one, not yet reached by the saturation front, having a water contents equal to the initial one (θ).

At time t, after beginning of the infiltration process, the cumulative infiltration F, that is the water volume which is infiltrated till that moment, can be express by the following formula:

$$F(t)(mm) = Kt + \Delta\theta\left(h_0 + \psi\right)\ln\left(1 + \frac{F(t)}{\Delta\theta\left(h_0 + \psi\right)}\right)$$

where:

K(m/h) = vertical permeability of the ground, usually sets equal to the 50% of the horizontal one;

t(h) = calculation time;

ψ(mm) = capillary rise;

h_0(mm) = hydraulic depth, in respect to the bottom of the lowered area.

Δθ = η-θ;

Since the parameter F appears in both the members of the equation, the solution has to be found through an iterative process, imposing a first value inside the second member, solving the equation and then substituting the new calculated value in the second member. Calculation has to be repeated until the difference between two consecutive values of F will be lower than a prefix limit (for example 0.001).

The value of the capillary rise may be chosen, selecting it by the following table:

Soil type	ψ(m)
Gravel	0.05-0.30
Coarse sand	0.30-0.80
Medium sand	0.12-2.40
Fine sand	0.30-3.50
Silt	1.5-12
Clay	>10

Known the cumulative infiltration, the potential infiltration ratio can be calculated by the following expression:

$$f(t)(mm/h) = K\frac{F(t) + \Delta\theta\left(h_0 + \psi\right)}{F(t)}$$

In precautionary way, it admits that the infiltration occurs only at the bottom of the lowered area

Sizing of draining trenches.

They are excavations with a narrow section having a permeable bottom.
Surface runoff, during the rainfall event, is conducted into the draining trench and then it's taken away through infiltration on the bottom . Sizing of draining trenches is usually performed through the estimate of its minimum volume, taking in account both the inflow water volumes and the outflow ones through infiltration.

Estimating the surface runoff.

Surface runoff can be calculated through the explained methods in the previous paragraphs, particularly using the rational and direct rainfall methods.

Estimating the potential infiltration ratio.

The procedure is the same seen in the case of green lowered surfaces, supposing that the infiltration occurs only at the bottom of the trench.

Sizing of infiltration wells.

They're wells having a circular cross-section with a permeable bottom. Surface runoff, during the rainfall event, is conducted into the infiltration wells and then it's taken away through infiltration on the bottom . Sizing of infiltration wells is usually performed through the estimate of its minimum volume, taking in account both the inflow water volumes and the outflow ones through infiltration.

Estimating the surface runoff.

Two cases are considered:

- constant surface runoff;
- variable surface runoff.

The first case usually happens when the wells are connected to a detention basin and the inflow comes from the discharge pipe. Vice versa the second case occurs when the surface runoff is directly carried to the infiltration wells.
In case of variable runoff, the calculation of the inflow volume can be performed through the methods seen in the previous paragraphs (rational method, direct rainfall method). The critical duration (t_r) of the rainfall cannot be directly calculated and thus it needs to proceed by trial
The total duration of the simulation may approximately be set equal to 2 times the rainfall duration.

Calculating the minimum volume of the wells.

Infiltration wells have to allow the infiltration of the surface runoff and the temporary storage of the water volume in excess until the end of the rainfall event.
Disregarding the evaporation, that's not significant during the rainfall event, the fundamental formula is the following:

$$\text{(a)}\ \left(Q_p - Q_f\right)\Delta t = \Delta W$$

where:

Q_p(mc/h) = surface runoff at time t (constant or variable);
Q_f(mc/h) = infiltration outflow at time t;
Δt(h) = calculation step;
ΔW(mc) = water volume inside the well.

The parameter Q_f is given by the following expression (Sieker, 1984):

$$\text{(b)}\; Q_f = \frac{k}{2}\left(\frac{L+h}{L+\frac{h}{2}}\right)A_f$$

where:

L (m) = water table depth measured from the bottom of the well;
h (m) = depth of the water column inside the well;
k(m/s) = permeability of the saturated subsoil;
A_f(mq) = draining area = $\pi(0{,}5d+0{,}5h)^2 - \pi d^2/4$
d(m) = well diameter

Finally the term ΔW is gotten by:

$$\text{(c)}\; \Delta W = A_p h$$

where:
Ap(mq) = area of the cross-section of the well = $\pi d^2/4$

Being the variable h, that's in both the members of the equation a), unknown, it has to proceed through an iterative way. Practically it fixes h=0, that is Q_f=0, for the first time step, and thus the a) is resolved. The first value of h is given by:

$$h = \frac{\Delta W_1}{A_p}$$

Using the calculated value of h , it's possible to estimate Qf, which will be then inserted in the formula c). The new value of h, at time $2\Delta t$, is given, resolving a) again.

$$\Delta W_2 = \Delta W_1 + \left(Q_p - Q_f\right)\Delta t$$

The process has to be repeated at least till a time equal to the duration of the rainfall.