# THEORETICAL BASIS.

# Probabilistic approach.

On the basis of this approach, the characteristic value of a geotechnical parameter is estimable using a probabilistic analysis applied to a set of measured data. Characteristic values are calculated as a function of a specific probability of not exceedance, usually 5 percent.

It's possible to recognize two different cases:

**1)Structures involving a large volume of subsoil**

Considering, for example, the angle of shearing resistance, the characteristic value of this parameter can be estimated through the following formula:

$$(1a)\ \varphi_k = \varphi_m - t_{n-1}^{0,05}\left(\frac{s_\varphi}{\sqrt{n}}\right)\ (n>5)$$

$$(1b)\ \varphi_k = \varphi_m - 1{,}645\left(\frac{\sigma_\varphi}{\sqrt{n}}\right)\ (n\leq 5)$$

dove:

| | |
|---|---|
| $\varphi_k$ | = characteristic value of the angle of shearing resistance; |
| $\varphi_m$ | = mean value of the angle of shearing resistance; |
| n | = number of measurements; |
| $\sigma_\varphi$ | = standard deviation of population (known by literature); |
| $s_\varphi$ | = standard deviation of the sample; |
| $t_{n-1}^{0,05}$ | =t by Student for n-1 degree of freedom and 5% probability of exceedance. |

The parameter $t_{n-1}^{0,05}$ can be interpolated through the following table.

| n-1 | $t_{n-1}^{0,05}$ |
|---|---|
| 1 | 6,314 |
| 2 | 2,920 |
| 3 | 2,353 |
| 4 | 2,132 |
| 5 | 2,015 |
| 6 | 1,943 |
| 7 | 1,895 |
| 8 | 1,860 |
| 9 | 1,833 |
| 10 | 1,812 |
| 11 | 1,796 |
| 12 | 1,782 |
| 13 | 1,771 |
| 14 | 1,761 |
| 15 | 1,753 |
| 16 | 1,746 |
| 17 | 1,740 |
| 18 | 1,734 |
| 19 | 1,729 |
| 20 | 1,725 |
| 21 | 1,721 |
| 22 | 1,717 |
| 23 | 1,714 |
| 24 | 1,711 |
| 25 | 1,708 |
| 26 | 1,706 |
| 27 | 1,703 |
| 28 | 1,701 |
| 29 | 1,699 |
| 30 | 1,697 |
| 40 | 1,684 |
| 60 | 1,671 |
| 120 | 1,658 |
| ∝ | 1,645 |

Intermediate values for n-1 can be gotten by linear interpolation. This procedure is obviously extendable to any kind of geotechnical parameter.

**2)Structures involving a small volume of subsoil.**

Taking into account again the angle of shearing strength φ, the characteristic value in this second case is determinable by the following formula.

$$(2)\,\varphi_k = \varphi_m(1+\chi V_\varphi)$$

dove:

$V_\varphi$ = coefficient of variation, given by the ratio between the standard deviation and the average of the φ values;

χ = parameter as a function of the used probabilistic distribution law and of the probability of not exceedance

Using a 5% probability of not exceedance, to which corresponds a χ value equal to –1.645, for a normal distribution, the relation (2) becomes:

$$(3)\,\varphi_k = \varphi_m(1-1.645V_\varphi)\,.$$

Thus, by a practical point of view, starting from a sample of values of φ, belonging to a specific subsoil layer, it has to calculate sequentially:

1. the mean value of φ($\varphi_m$);
2. the standard deviation of φ($\sigma_\varphi$);
3. the C.O.V. ($V_\varphi=\sigma_\varphi/\varphi_m$);
4. the characteristic value of φ ($\varphi_k$).

Obviously, it can't always to collect a sufficient number of measurements to apply in a rigorous way the previously illustrated method. In these cases it's possible however to use COV values obtained by the scientific literature. Cherubini, Giasi e Rethati (1993) suggest the following values of COV for some geotechnical parameters:

| Parameter | Average | Standard deviation | C.O.V. |
|---|---|---|---|
| φ | 0.1219 | 0.0615 | 0.5045 |
| $c_u$ | 0.4324 | 0.2328 | 0.5384 |
| γ | 0.0685 | 0.0359 | 0.5241 |
| $C_c$ | 0.3551 | 0.1269 | 0.3574 |

Looking at this table, it can notice the low value of the γ (unit weight) parameter. This means that it's possible to use directly the mean value instead of the characteristic value.

# Geotechnical approach.

On the basis of this approach, the characteristic value of a geotechnical parameter is estimable in respect of the strain level imposed by the considered limit state. In the case of the ultimate limit state it should be used the angle of shearing strength and the cohesion in post-peak condition, that corresponding to the high-strain level immediately following the collapse of the soil.

In case of the angle of shearing strength, the post-peak value (or at constant volume $\varphi_{cv}$) can be estimated approximately by the Bolton relation (1986):

$$\varphi_{cv} = \varphi_{picco} - 3\left( D_r \ln \frac{22}{\sigma'} - 1 \right);$$

where Dr is the relative density and σ' is the vertical effective lithostatic pressure.
The following table, processed by Hough (1957), permits to compare $\varphi_{picco}$ and $\varphi_{cv}$ for several lithologies:

| Litology | Min $\varphi_{cv}$ | Max $\varphi_{cv}$ | Min $\varphi_{peak}$ | Max $\varphi_{peak}$ |
|---|---|---|---|---|
| Silt | 26 | 30 | 28 | 32 |
| Homogeneous sand from medium to fine | 26 | 30 | 30 | 34 |
| Well graded sand | 30 | 34 | 34 | 40 |
| Sand and gravel | 32 | 36 | 36 | 42 |

In case of undrained conditions, where $\varphi$=0 e $c_u$>0, it's possible to use conservatively the Mesre relation (1975):

$$c_u = 0{,}22\sigma_v OCR$$

where $\sigma_v$ is the vertical effective lithostatic pressure and OCR is the overconsolidation ratio of the layer.