PROGRAM GEO – Sisma ver.4

SUMMARY

Theoretical basis

Estimating the design earthquake

Probabilistic analysis: Cornell method

The calculation procedure requires first an estimation of the frequency-magnitude relationship (Gutenberg & Richter) inside the seismogenic zone in which the examined site lies. It proceeds in the following way.
1)Execute the extraction of the seismic events, from the earthquake catalogue, which lie inside the reference area.
2) Set a magnitude interval for the calculation, selecting an appropriate lower threshold value (m0), for example 4, and a higher threshold value (m1), taking the maximum magnitude value found in the seismogenic area and increasing it by 0.3-0.4. Then, if the maximum recorded magnitude inside the seismogenic area is 6, insert a higher threshold value equal to 6.3-6.4.
3) Divide the m1-m0 interval in an appropriate number of subintervals having a size of 0.3-0.5 and, then, for every subinterval, estimate the frequency of the events through the formula:

$$\nu(M) = \frac{N}{Tempo}$$

where the variable Tempo stands for the number of years including in the earthquake catalogue. So, if, for example, the extract from the catalogue includes earthquakes since the year 1200 to the year 2000 and, for a magnitude higher than 4, the recorded events are 75, the frequency is:

$$\nu(M \geq 4) = \frac{75}{(1992 - 1200)} = 0{,}0947$$

4) Proceed, calculating the frequencies relative to all the subintervals of magnitude taken into account ($\nu(M\geq4)$, $\nu(M\geq4{,}4)$, $\nu(M\geq4{,}8)$...), drawing it on a chart, having along the X axis the magnitude values and along the Y axis the decimal logarithm of the frequency.

5) Finally, interpolate the data through the method of least squares, to get the variables *a* and *b* of the interpolating curve:

$$\log \nu = a - bM$$

which represents the Gutenberg-Richter relationship of the frequency as a function of magnitude.

Reminding that frequency is the reciprocal of time, the Gutemberg-Ricther relationship can be used to get a magnitude value as a function of a specific return period.

Cornell method is based on a statistical model of poissonian type, founded on the hypothesis that every single seismic event be independent by the others occurred in the past. As to the location of seismic source, it supposes it can lie in whatever spot inside the seismogenic area. The probability that a specific critical value of the seismic acceleration A be not exceeded, on annual basis, at the reference site is given by the following formula, on the hypothesis of a point-shaped seismic source:

$$(1)\, P[a \le A] = \exp\left[-\nu F C \exp\left(-\frac{\beta A}{b_2}\right)\right]$$

where:

ν = annual frequency rate;

β = b ln(10); where b is the variable of the Gutenberg-Richter relationship;

F $=r^{-\beta c3/c2}$; where r is the distance of the seismic source from the reference site, given by: $r^2=R^2+h^2$ with R=epicentral distance e h=ipocentral depth;

C $=\exp\left[\beta\left(\frac{c1}{c2}+m0\right)\right]$

m0 = lower threshold of magnitude in the Gutenberg-Richter relationship.

Variables c1, c2 and c3 depend on the selected attenuation relationship, having the following form:

$$\ln(a_{max}) = c_1 + c_2M - c_3\ln(R^2 + h^2)$$

In the following table, values of c1, c2, c3 and h, relative to some most used attenuation relationships are displayed.

Method	c1	c2	c3	h
Pugliese and Sabetta (1987) *stiff and deep soil*	-3.2075	0.8358	0.5	5.0
Pugliese and Sabetta (1987) *shallow soil*	-4.2483	0.8358	0.5	5.0
Cauzzi and Faccioli (2008) *EC8 class A*	-5.2676	1.2802	0.791	5.0
Cauzzi and Faccioli (2008) *EC8 class B*	-4.7610	1.2802	0.791	5.0
Cauzzi and Faccioli (2008) *EC8 class C*	-4.5676	1.2802	0.791	5.0
Cauzzi andFaccioli (2008) *EC8 class D*	-4.5031	1.2802	0.791	5.0
Ambraseys et al.(1996) *rock*	-3.4078	0.6125	0.461	3.5
Ambraseys et al.(1996) *stiff soil*	-3.1384	0.6125	0.461	3.5
Ambraseys et al.(1996) *soft soil*	-3.1223	0.6125	0.461	3.5
Ambraseys et al.(1996) *very soft soil*	-2.8529	0.6125	0.461	3.5
Wang et al. (1999)	0.9901-0.01105R-6.8886	0.9855	0.382	0.0

The formula (1) is valid only in case of point-shaped source. In order to take in account the contribution of a circular surface, having a total area S and a radius R_0, centred on the examined site, the probability of not exceeding a specific value of seismic acceleration takes the following form:

$$P\left[a \leq a_g\right] = \exp\left\{ -\frac{\nu}{E \cdot A}\left[C \cdot G \cdot \exp\left(-\beta \frac{a_g}{c_2}\right) - (1 - E)A\right]\right\}$$

where:

$$E = 1 - \exp[-\beta(m_1 - m_0)]$$

$$A = R_0^2$$

$$G = \frac{1}{\gamma}\left(R_0^{\,2} + h^2\right)^{\gamma}$$

The return period of the reference event is given by:

$$T = \frac{1}{1 - P\left[a \leq a_g\right]}$$

Historical-statistical analysis

It's based on the construction of a frequency-acceleration relationship, similar to Gutenberrg-Richter one relative the seismic magnitude.

1)Execute the extraction of the seismic events, from the earthquake catalogue, which lie inside the reference area. Estimate then, applying an appropriate attenuation relationship, the acceleration value endured in the reference site for every seismic event extracted from the catalogue.
2)Set a calculation interval of acceleration, selecting an appropriate lower threshold value (a0), for example 0.03, and a higher threshold value (a1), taking in account the maximum recorded value of acceleration and increasing it by 0.01.
3) Divide the a1-a0 interval in an appropriate number of subintervals and, then, for every subinterval, estimate the frequency of the events through the formula:

$$\nu(a_g) = \frac{N}{Tempo}$$

where the variable Tempo stands for the number of years including in the earthquake catalogue.
4)Proceed, calculating the frequencies relative to all the subintervals of acceleration taken into account, drawing it on a chart, having along the X axis the acceleration values and along the Y axis the decimal logarithm of the frequency.
5)Finally, interpolate the data through the method of least squares, to get the variables *a* and *b* of the interpolating curve:

$$\log(\nu) = a - b \cdot a_g$$

Reminding that frequency is the reciprocal of time, the relationship can be used to get an acceleration value as a function of a specific return period.
Respect to Cornell method, the historical-statistical one tends to be less conservative in case of centuries long return periods.

Seismic parameters through analysis of a accelerogram

Beside the ground peak acceleration (PGA), severity of a seism can be estimated, calculating a set of additional parameters directly correlable to the damage level observed in the historical earthquakes.

- ❑ Arias intensity

The Arias intensity(1970) represents an index proportional to the seismic released energy. It's the integral of the square of the recorded seismic acceleration extended to the whole duration T of the accelerogram .

$$Ia = \frac{\pi}{2g}\int_o^T a^2(t)dt$$

- ❑ Saragoni factor

It's defined as the ratio between the Arias intensity and the square of the number of times in which the seismic signal crosses the zero-amplitude line per unit time (ν_0):

$$P_D = \frac{Ia}{\upsilon_0^{\ 2}}$$

- Significant duration

It's the interval of time between 5% and 95% of the total Arias intensity.

- Cumulative absolute velocity

It's the integral of the absolute values of acceleration extended to the total duration T of the accelerogram:

$$CAV = \int_o^T |a(t)| dt$$

- Cumulative absolute displacement

It's the integral of the absolute values of velocity extended to the total duration T of the accelerogram:

$$CAD = \int_o^T |v(t)| dt$$

- Fajfar index

It's defined as the product between the peak velocity and the significant duration t_d raised to 0.25

$$F_i = PGVt_d^{\ 0.25}$$

Seismic characterization of the site.

Soil characterization

In order to classify the reference site by a seismic point of view it's necessary to know the stratigraphic characteristics of the ground. Particularly they have to be known:

- number and thickness of the soil layers lying above the bedrock or the bedrock-like, that are a rocky substratum (bedrock) or a soil stratum (bedrock-like) having a shear wave velocity clearly higher than the shallow layers (generally Vs≥500-800 m/s);
- the shear wave velocity of the shallow layers.

Characterization can be performed through geophysical surveys (seismic refraction, downhole, crosshole, seismic cone, MASW, ReMi, HVSR etc.) or, in absence of these, through penetrometric tests, both CPT and SPT.

Soil charecterization by SPT

❑ Otha e Goto (1978)

The Otha and Goto (1978) formula, suggested by the TC4 manual for the zonation of the geotechnical hazards, correlates Nspt to Vs, taking in account both the soil layer age and its prevalent granulometry. It has the following expression:

$$V_s(m/s) = 68 {N_{spt}}^{0.17} D^{0.2} EF$$

where D(m) is the average depth to the layer from the surface ground, E is a correction factor which takes in account of the soil layer age (Table I) and F is a correction factor as a function of the prevalent granulometry (Table II).

Soil layer age	E factor
Holocene	1.0
Pleistocene	1.3

Table I

Granulometry	F factor
Clay	1.00
Fine sand	1.09
Medium sand	1.07
Coarse sand	1.14
Gravelly sand	1.15
Gravel	1.45

Table II

- Imai (1977)

The Imai formula, valid for sandy and clayey layers, has the following form:

$$V_s\,(m/s) = \alpha N_{spt}{}^{\beta}$$

where α and β have the following values:

Lithology and age	α	β
Clay Holocene	102	0.29
Sand Holocene	81	0.33
Clay Pleistocene	114	0.29
Sand Pleistocene	97	0.32

- Lee (1990)

It's valid for holocenic soil layers, having a granulometry between clay and sand, and has the following form:

$$V_s\,(m/s) = \alpha N_{spt}{}^{\beta}$$

where α and β have the following values:

Lithology	α	β
Clay	114.4	0.31
Silt	105.6	0.32
Sand	57.4	0.49

- Muzzi (1984)

This formula is valid for holocenic sandy and clayey soil layers. It has the following form:

$$V_s\,(m/s) = \alpha N_{spt}{}^{\beta}$$

where α and β have the following values:

Lithology	α	β
Clay	102	0.292
Sand	80.6	0.331

Soil charecterization by CPT

- Barrow e Stokoe (1983)

This expression is valid for all the soil types and has the following form:

$$V_s\,(m/s) = \alpha + \beta q_c$$

where α and β have the values, respectively, of 50.6 and 2.1 and q_c is expressed as kg/cmq.

- Mayne e Rix (1995)

This formula, valid for cohesive soil layers, has the following form:

$$V_s\,(m/s) = \alpha q_c{}^{\beta}$$

where α and β have the values, respectively, of 1.75 and 0.627 and q_c is expressed as kPa.

Seismic site classification

Method based on the rigidity of the soil layers above the bedrock.

Suggested by Draft (1989) and adopted in the Chinese Aseismic Design Code for Structures, this classification is based on estimating a parameter, the site index, as a function of the mean shear modulus and of the thickness of the soil layers lying above the bedrock.
The mean shear modulus is calculated through the following relation:

$$G(kPa) = \frac{\sum_{i=1}^{n} h_i \frac{\gamma_i}{9.81} V_{si}^{2}}{\sum_{i=1}^{n} h_i}$$

where:
h(m) = thickness of the i-th layer;
γ(kN/mc) = unit weight of the i-th layer;
V_s (m/s) = shear wave velocity of the i-th layer;
n = number of soil layers above the bedrock.
If the total thickness of the soil layers above the bedrock exceeds 20 m, they have to take in account only those lying up to this depth.
According to this method, it has to define as bedrock or bedrock-like whatever layer having S wave velocity higher than 500 m/s.
The site index is calculated by the following formula:

$$\mu = 0.6\mu_g + o.4\mu_h$$

where μ_g is the contribution of the mean shear modulus to the site index, given by this relation:

$$\mu_g = 1 - \exp\left[-0.66(G - 30000)10^{-5}\right] \qquad \text{if G>30000 kPa;}$$

$$\mu_g = 0 \qquad \text{In the other case;}$$

and μ_h is the contribution due to the total soil layer thickness and it is given by:

$$\mu_h = \exp\left[-0.916(H-5)^2 10^{-2}\right]$$

$\mu_h = 0$ if H>80 m

$\mu_h = 1$ if H≤5 m

where H is hte total thickness of the soil layers above the background.
In case it is both G>500000 kPa and H≤5 m, it's necessary to set $\mu_h=\mu_g=1$.
The site classification is given by the following table:

Type	Stiff	Med.stiff	Med. soft	Soft
Site index	1>μ>0.9	0.9>μ>0.3	0.3>μ>0.1	0.1>μ>0

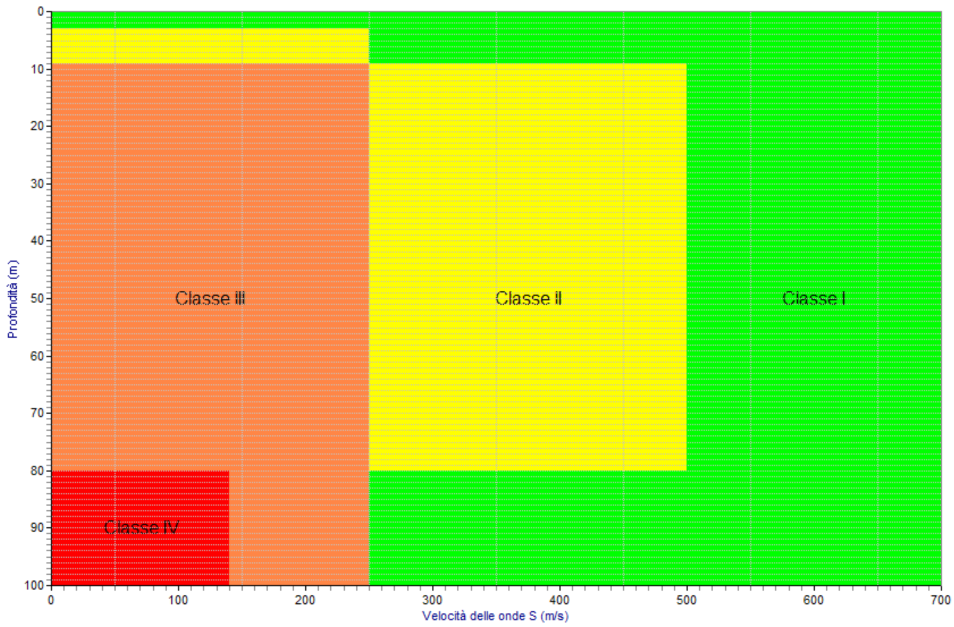
The seismic site amplification tends to increase at the decreasing of the site index.

Method based on the shear wave velocity of the soil layer overlying the bedrock.

Suggested by the Chinese Aseismic Design Code for Structures, this method involves a site classification based on both the mean shear wave velocity and the total thickness of the soil layers overlying the bedrock.
The average share wave velocity is calculated as harmonic mean of the Vs of the single layers lying above the bedrock.

$$V_{sH} = \frac{H}{\sum_{i=1,N} \frac{h_i}{V_{si}}}$$

The total thickness of the soil layers overlying the bedrock (H) is calculated starting by the top of the first layer having a S wave velocity higher than 500 m/s. In the following scheme are displayed the four site classes contemplated by the method.



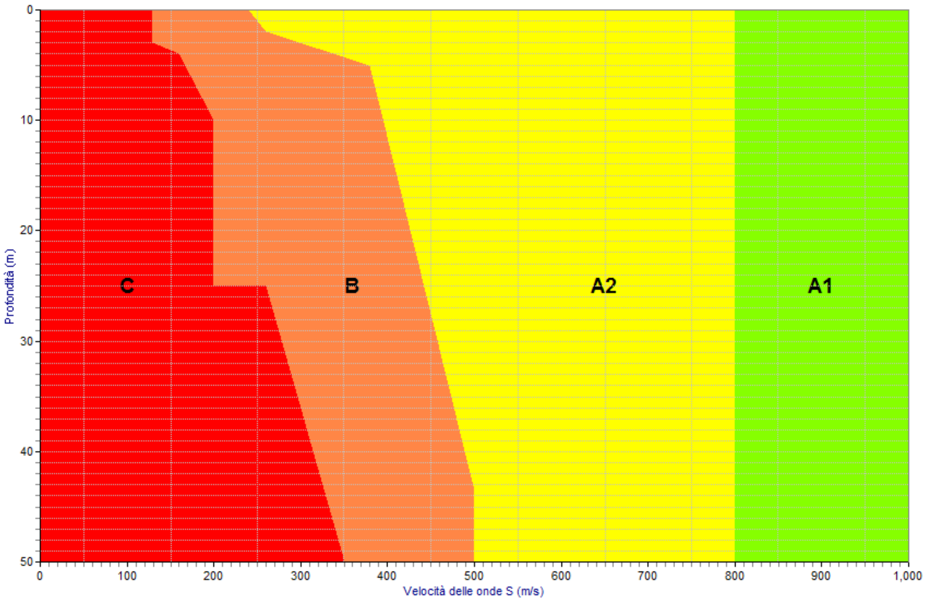
The site class is given, intersecting the depth of the bedrock along the Y axis with the mean Vs along the X axis.
The seismic site amplification tends to increase passing from the class I to the class IV.

Method suggested by Eurocode 8 (1994).

In EC8, 1994 version, is contemplated a site classification as a function of both the average share wave velocity of the soil layers overlying the bedrock and their total thickness.
The average Vs is calculated by the following formula:

$$V_{sH} = \frac{H}{\sum_{i=1,N} \frac{h_i}{V_{si}}}$$

Three classes are identified, A (divided in two subclasses), B and C, each of them is associated to a specific elastic response spectrum.
The reference scheme to determine the site class is the following:



The site class is given, intersecting the depth of the bedrock along the Y axis with the mean Vs along the X axis.
The seismic site amplification tends to increase passing from the class A1 to the class C.

Method suggested by Eurocode 8 (2003).

Five classes are identified, A, B, C, D and E, each of them is associated to a specific elastic response spectrum. The reference scheme is the following:

Ground type	Description of stratigraphic profile	Parameters		
		$v_{s,30}$ (m/s)	N_{SPT} (blows/30cm)	c_u (kPa)
A	Rock or other rock-like geological formation, including at most 5 m of weaker material at the surface	> 800	–	–
B	Deposits of very dense sand, gravel, or very stiff clay, at least several tens of m in thickness, characterised by a gradual increase of mechanical properties with depth	360 – 800	> 50	> 250
C	Deep deposits of dense or medium-dense sand, gravel or stiff clay with thickness from several tens to many hundreds of m	180 – 360	15 - 50	70 - 250
D	Deposits of loose-to-medium cohesionless soil (with or without some soft cohesive layers), or of predominantly soft-to-firm cohesive soil	< 180	< 15	< 70
E	A soil profile consisting of a surface alluvium layer with v_s values of type C or D and thickness varying between about 5 m and 20 m, underlain by stiffer material with v_s > 800 m/s			

The parameter V_{s30} is the weighted average of the S wave velocity of the soil layers overlying the bedrock to the depth of up to 30 m starting from the ground surface. It's given by the following relation:

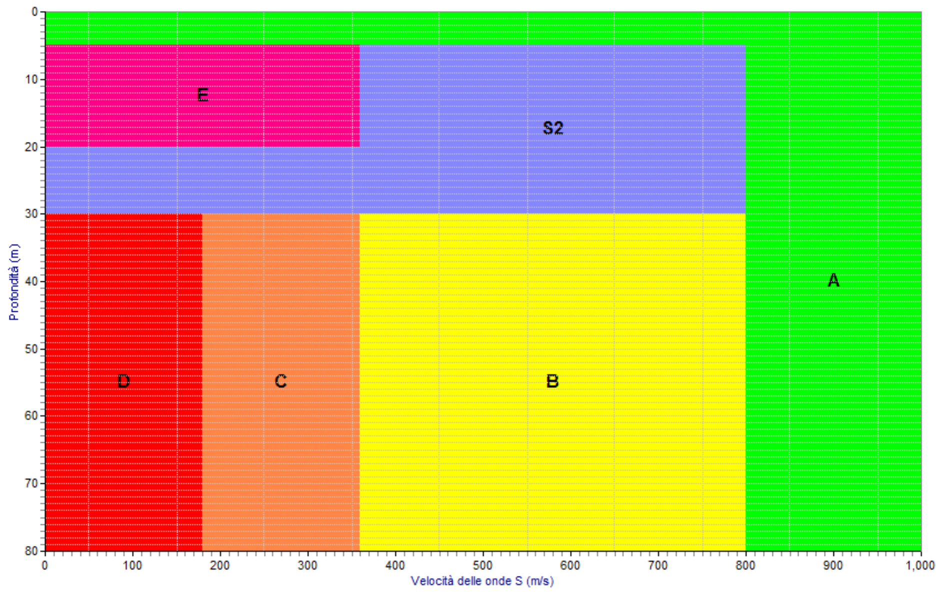
$$V_{s30} = \frac{30}{\sum_{i=1,N} \frac{h_i}{V_{si}}}$$

In the same way for N_{spt30} and cu_{30}:

$$N_{spt,30} = \frac{30}{\sum_{i=1,N} \frac{h_i}{N_{spt,i}}}$$

$$cu_{30} = \frac{30}{\sum_{i=1,N} \frac{h_i}{cu_i}}$$

If the Vs data in the first 30 m are not available and the soil layers are composed by a succession of cohesive and granular layers, it proceeds estimating the site classes both through Nspt,30 and cu30, assuming then the worst of them.
The site class is given, intersecting the depth of the bedrock along the Y axis with the mean Vs along the X axis.
The seismic site amplification tends to increase passing from the class A to the class D.



In addition to the five standard classes, EC8 gives other two which requires specific and in-depth analysis to define the seismic site effects.

S_1	Deposits consisting – or containing a layer at least 10 m thick – of soft clays/silts with high plasticity index (PI > 40) and high water content	< 100 (indicative)	_	10 - 20
S_2	Deposits of liquefiable soils, of sensitive clays, or any other soil profile not included in types A – E or S_1			

Estimating the fundamental period of resonance of the soil.

The fundamental period of resonance of the soil T can be directly measured, for example, through a HVSR survey, or estimated by empirical correlations.

- ❑ Empirical correlation based on the weighted average of the shear wave velocities.

The T value can be evaluated, passing through the calculation of the weighted average of Vs inside the soil layers overlying the bedrock:

$$\overline{V_s} = \frac{\sum_{i=1,N} V_{si} h_i}{H} \qquad T = \frac{4H}{\overline{V_s}}$$

where H is the total thickness of the soil layers above the bedrock and Vs_i e h_i , respectively, the S wave velocity and the thickness of the i-th layer.

- ❑ Empirical correlation based on the weighted average of rigidity and density.

The T value can be estimated, passing through the calculation of the weighted average of G0 (low strain shear modulus) and of ρ (mass density of the layer, given by the ratio between the unit weight and the acceleration of gravity g: ρ=γ/g))inside the layers overlying the bedrock.

$$\overline{G_0} = \frac{\sum_{i=1,N} G_{0i} h_i}{H} \quad \overline{\rho} = \frac{\sum_{i=1,N} \rho_i h_i}{H} \quad T = \frac{4H}{\sqrt{\frac{\overline{G_0}}{\overline{\rho}}}}$$

where H is the total thickness of the soil layers.

- Empirical correlation based on the sum of the natural periods.

The T value can be estimated, calculating the sum of the fundamental periods of resonance of every soil layer above the bedrock:

$$T = \sum_{i=1,N} 4 \frac{h_i}{V_{si}}$$

where H is the total thickness of the soil layers.

- Empirical correlation based on the simplified approach by Rayleigh.

The relation is the following:

$$\omega = \sqrt{\frac{4 \sum_{i=1,N} \frac{(H - Z_i)^2}{V_{si}^{\ 2}} h_i}{\sum_{i=1,N} (X_i + X_{i+1})^2 h_i}} \quad T = \frac{2\pi}{\omega}$$

where:

$X_{i+1} = X_i + \frac{H - Z_i}{V_{si}^{\ 2}} h_i$ e $X_1 = 0$, being H-Z_i the mean depth of the i-th layer.

Seismic amplification effects.

Geomorphological and stratigraphic local factors can modify the characteristics of the seismic motion, filtering the waves during their travel from the bedrock to the ground surface. The filtering effect leads to a redistribution of the energy, implicating an amplification of the vibrational motion associated to some frequencies.
Several empirical methods to estimate the seismic amplification are known in the scientific literature. They are based on the stratigraphic characteristics of the ground and on the evaluation of the shear wave velocity of the soil layers overlying the bedrock. According to the definition given by the International Manual TC4, they are methods of analysis of II level.

Stratrigraphic effects: II level methods.

Midorikawa method (1987)

It is a method suggested by the TC4 manual. The amplification factor relative to the peak ground acceleration (PGA) is given by the following relation:

$$F_a = 68 V_s^{-0.6} \text{ per } V_s < 1100 \text{ m/s}$$

$$F_a = 1 \text{ per } V_s \geq 1100 \text{ m/s}$$

where V_s is the weighted average of the S wave velocity to a maximum depth of up to 30 m from the ground surface. If the bedrock, defined by this method as the first layer, having a shear wave velocity equal or higher than 1100 m/s, is deeper than 30 m, they have to be considered only the soil layers between 0 and 30 m.
The peak acceleration at the ground surface is given by:

$$PGA(g) = a_{bedrock} F_a$$

wher $a_{bedrock}$ is the seismic acceleration at the bedrock.

Boore and Atkinson method (2008)

The amplification factors relative to the peaks of acceleration and velocity are given by:

$$Fa = F_L + F_{NL}$$

where:
F_L = linear term of the amplification factor;
F_{NL} = nonlinear term of the amplification factor e.

The linear term is given by the following relation:

$$F_L = b_{lin} \ln(V_{s30}/V_{ref})$$

where:
b_{lin} = coefficient equal to -0.360 (PGA) and to -0.600 (PGV);
V_{ref} = reference velocity of the S waves (bedrock); Boore and Atkinson set V_{ref}=760 m/s
V_{s30}= weighted average of the S wave velocities in the soil layers overlying the bedrock to a depth of up to 30 m.

The nonlinear term, which to a first approximation can be neglected, is given by:

(a) $pga4nl \le a_1$:

$$F_{NL} = b_{nl} \ln(pga\_low / 0.1)$$

(b) $a_1 < pga4nl \le a_2$:

$$F_{NL} = b_{nl} \ln(pga\_low / 0.1) + c[\ln(pga4nl / a_1)]^2 + d[\ln(pga4nl / a_1)]^3$$

(c) $a_2 < pga4nl$:

$$F_{NL} = b_{nl} \ln(pga4nl / 0.1)$$

where
pga4nl = seismic acceleration at the bedrock;
a_1 = 0.03 g acceleration threshold for the linear amplification;
a_2 = 0.09 g acceleration threshold for the nonlinear amplification;
pga_low = 0.06 g threshold between linear and nonlinear amplification;
b_{nl} = coefficient for nonlinear term, given by:

(a) $V_{S30} \le V_1$:

$$b_{nl} = b_1 .$$

(b) $V_1 < V_{S30} \le V_2$:

$$b_{nl} = (b_1 - b_2) \ln(V_{S30} / V_2) / \ln(V_1 / V_2) + b_2 .$$

(c) $V_2 < V_{S30} < V_{ref}$:

$$b_{nl} = b_2 \ln(V_{S30} / V_{ref}) / \ln(V_2 / V_{ref}) .$$

(d) $V_{ref} \le V_{S30}$:

$$b_{nl} = 0.0 .$$

where:

V_1 = 180 m/s;
V_2 = 360 m/s;
b_1 = -0.640 (PGA) and -0.500 (PGV;)
b_2 = -0.14 (PGA) and-0.06 (PGV).

Finally coefficients *c* and *d* are given by:

$$c = (3\Delta y - b_{nl}\Delta x) / \Delta x^2$$

$$d = -(2\Delta y - b_{nl}\Delta x) / \Delta x^3$$

where:

$$\Delta x = \ln(a_2 / a_1)$$

$$\Delta y = b_{nl} \ln(a_2 / pga\_low).$$

Stratrigraphic effects: III level methods.

1D propagation model of S waves into a horizontal layered subsoil.

Vertical propagation of shear waves, with frequency ω, causes horizontal displacements u(z,t), which must satisfy the equation:

$$(1)\,\rho\frac{\partial^2 u}{\partial t^2} = G\frac{\partial^2 u}{\partial z^2} + \eta\frac{\partial^3 u}{\partial z^2 \partial t}$$

Harmonic displacements *u* with frequency ω, can be expressed in the following form too:

$$(2)\,u(z,t) = U(z)e^{i\omega t}$$

Substituting (2) into (1):

$$(3)\,(G + i\omega\eta)\frac{\partial^2 U}{\partial z^2} = \rho\omega^2 U$$

where ρ is the mass density and G is the shear modulus of the soil layer.
Equation (3) has the following general solution:

$$(4)\,U(z) = Ee^{ikz} + Fe^{-ikz}$$

where:

$$k = \sqrt{\frac{\rho\omega^2}{G^*}}$$

G* is the complex shear modulus:

$$G^* = G(1 + 2i\beta)$$

and β is the critical damping factor:

$$\beta = \frac{\omega\eta}{2G}$$

In equation (4) *E* is the incident wave traveling upwards and *F* is the reflected wave traveling downwards.
Combine (2) and (4):

$$(5)\, u(z,t) = \left(Ee^{ikz} + Fe^{-ikz}\right)e^{i\omega t}$$

In a multilayer subsoil, at the top of the layer *n,* with thickness *h,* the displacements are:

$$(6)\ u_n(z=0) = \left(E_n + F_n\right)e^{i\omega t}$$

at the bottom:

$$(7)\ u_n(z=h) = \left(E_n e^{ik_n h_n} + F_n e^{-ik_n h_n}\right)e^{i\omega t}$$

Shear stresses acting along the horizontal plane at the top and bottom of the layer are respectively:

$$(8)\, \tau_n(z=0) = ik_n G_n^{*}\left(E_n + F_n\right)e^{i\omega t}$$

$$(9)\, \tau_n(z=h) = ik_n G_n^{*}\left(E_n e^{ik_n h_n} + F_n e^{-ik_n h_n}\right)e^{i\omega t}$$

Shear strains ($\gamma(z,t) = \frac{\partial u}{\partial t}$) are instead:

$$\gamma_n(z=0) = ik_n\ \left(E_n + F_n\right)e^{i\omega t}$$
$$\gamma_n(z=h) = ik_n\left(E_n e^{ik_n h_n} + F_n e^{-ik_n h_n}\right)e^{i\omega t}$$

In a multi-layer subsoil the parameters ρ, G e β change generally with the depth. Thus E and F have different values. Stresses and displacements must be continuous at all interfaces. Amplitudes E and F of the incident and reflected wave of the layer *n* can be expressed in the following way:

$$(10)\, E_{n+1} = \frac{1}{2} E_n (1+\alpha_n) e^{ik_n h_n} + \frac{1}{2} F_n (1-\alpha_n) e^{-ik_n h_n}$$

$$(11)\, F_{n+1} = \frac{1}{2} E_n (1-\alpha_n) e^{ik_n h_n} + \frac{1}{2} F_n (1+\alpha_n) e^{-ik_n h_n}$$

where:

$$\alpha_n = \sqrt{\frac{\rho_n G_n^*}{\rho_{n+1} G_{n+1}^*}}$$

is the complex impedance ratio.
At the ground surface the shear stress has to be zero. From equation (8):

$$E_1 = F_1.$$

For the case $E_1=F_1=1$, the amplitudes E and F of the layer *n* can be determined by substituting this condition in the formulas (10) and (11) , starting from the ground surface to the bedrock.
Transfer function between the displacements at level *n* and *n+1* is defined by:

$$(12)\, A_{n+1,n}(\omega) = \frac{u_n}{u_{n+1}} = \frac{E_n + F_n}{E_{n+1} + F_{n+1}}$$

At the bedrock interface E'=F' (shear stress=0). As a result transfer function of the shear wave at the ground surface in respect to the bedrock is given by:

$$(13)\; A_{bedrock,1}(\omega) = \frac{1}{E_{bedrock}}$$

Evaluation of the seismic motion on the ground surface.

The calculation procedure seen in the previous paragraph is referred to the case of a single wave with angular frequency ω.
An accelerogram is composed by a set of acceleration values *a* inside a interval of recording time T. The *a* values are sampled with a constant time step Δt, sampling interval, usually 0.01 or 0.02 seconds. Thus the duration of the earthquake is given by:

$$(14)\, T(s) = n\Delta t$$

where *n* is the total number of the sampling intervals.
A seismic pulse, recorded as an accelerogram, can be considered as a finite sum of waves having different frequency ω and thus can be expressed in the form of a discrete Fourier transform.

$$(15)\, a(t) = \sum_{j=0}^{j=n/2} \left(a_j e^{i\omega_j t} + b_j e^{-i\omega_j t} \right)$$

where n/2 are the angular frequencies given by:

$$\omega_j = \frac{2\pi}{n\Delta t} j$$

The variables *a* and *b*, as a function of ω, known as complex coefficients, are gotten through the following formulas:

$$(16)\, a_j = \frac{1}{n} \sum_{k=1}^{n-1} a(t) e^{-i\omega_j t}$$

$$(17)\, b_j = \frac{1}{n} \sum_{k=1}^{n-1} a(t) e^{i\omega_j t}$$

Practically, the values of *a* and *b* are calculated, using the fast Fourier transform (FFT), and graphically displayed as Fourier's spectra. Particularly, the amplitude spectrum is given by the following relation:

$$(18) \left|F(\omega)\right|_j = \sqrt{{a_j}^2 + {b_j}^2}$$

The calculation of the amplification factor, for each values of ω, permits to generate the transfer function of the seismic motion from the bedrock to the ground surface. If the formula (15) refers to the recorded accelerogram on the bedrock, the one at the ground surface is given by the following expression:

$$(19)\, a_{superficie}(t) = \sum_{j=0}^{j=n/2} A_{bedrock,1}(\omega_j)\left(a_j e^{i\omega_j t} + b_j e^{-i\omega_j t}\right)$$

The $a_{superficie}$ values, throughout the duration T of the earthquake, are computed using the inverse fast Fourier transform (IFFT) applied to the (19).

Evaluation of the stratigraphic seismic amplification through an equivalent linear model

The calculation procedure illustrated in the previous paragraph is applicable in case of nonlinear behavior of the subsoil too. A nonlinear behavior usually arises in case of earthquake with peak acceleration higher than 0.1 g. Both the shear modulus G and the damping factor β of the subsoil change as a function of the strain level due to the seismic stress. This phenomenon can be taking in account using a linear equivalent model. The procedure is the following:

- A first seismic response is calculated inserting G_0 and β_0, the shear modulus and the damping factor for low strain, of every soil layer.
- The shear strain, due to the seism, is evaluated; then G and β are updated on the basis of the G(γ) and β(γ) curves (see the next paragraph).
- The steps 1 and 2 have to be repeated till when the difference between the shear strains calculated in two consecutive iterations is below a predetermined limit.

Evaluation of the G(γ) and β(γ) curves.

The G(γ) and β(γ) curves are usually gotten through laboratory tests (resonant column test). As an alternative it's possible to use the empirical curves, as a function of the lithology, suggested by several authors.

*Clay (*Seed & Idriss)

γ (%)	G/G_0	β (%)
0.0001	1	0.24
0.0003	1	0.42
0.001	1	0.8
0.003	0.981	1.4
0.01	0.941	2.8
0.03	0.847	5.1
0.1	0.656	9.8
0.3	0.438	15.5
1	0.238	21
3	0.144	25
10	0.11	28

*Sand (*Seed & Idriss)

γ (%)	G/G_{max}	β (%)
0.0001	1	0.24
0.0003	1	0.42
0.001	0.99	0.8
0.003	0.96	1.4
0.01	0.85	2.8
0.03	0.64	5.1
0.1	0.37	9.8
0.3	0.18	15.5
1	0.08	21
3	0.05	25
10	0.035	28

*Rock (*Seed e Idriss)

γ (%)	G/G_{max}	β (%)
0.0001	1	0.4
0.0003	1	0.8
0.001	0.9875	1.5
0.003	0.9525	3
0.01	0.9	4.6
0.03	0.81	
0.1	0.725	
1	0.55	

*Gravel and pebbles-Gravel and sand (*Rollins)

γ (%)	G/G_{max}	β (%)
0.0001	1	1
0.0003	1	1
0.001	0.96	1.5
0.003	0.88	2.1
0.01	0.75	4
0.03	0.55	7
0.1	0.33	11
0.3	0.19	14
1	0.05	17

*Gravelly clay-Silty clay(*Rollins)

γ (%)	G/G_{max}	β (%)
0.0001	1	2
0.0003	1	2
0.001	0.97	2.1
0.003	0.95	2.2
0.01	0.88	3
0.03	0.70	4.5
0.1	0.37	10
0.3	0.15	17
1	0.05	25

Topografic effects: II level methods.

Method suggested by Eurocode 8 (2003).

The topographic amplification factor St is directly given by the following table:

Description	Building location	Topographic amplification factor S_T
flat or average slope angles of less than ~ 15°	--	1.0
isolated cliffs and slopes	near the top edge	≥ 1.2 [1)]
ridges with crest widths significantly less than the base width	near the top of the slope	≥ 1.4 [1)] for angles greater than 30°
		≥ 1.2 [1)] for angles 15° to 30°
The value of S_T may be assumed to decrease as a linear function of the height above the base of the cliff or ridge, and to be unity at the base. [1)] Increase by 20% in presence of a loose surface layer.		

The parameter St has a value equal to 1 at the bottom of the slope and reaches the maximum value on the top, increasing in a linear way. It has to be applied only in case of slope having an height of 30 m or more.

Simplified elastic response spectra.

Calculation of the elastic response spectra.

The seismic response of a building is often simplified using a single degree-of-freedom system (SDOF). Such an oscillator, staying on the ground in an initial condition of stillness [y(0)=0], and then forced to move by an earthquake, will be subject to a displacement y, as a function of the time, given by :

$$y(t) = -\frac{1}{\omega_d}\int_0^t a(\tau)e^{-\zeta\omega_n(t-\tau)}sen[\omega_d(t-\tau)]d\tau$$

where the integral at the second member is known as Duhamel's integral and where:

ζ = damping factor of the oscillator, normally lower than 0.1 (10 %) and usually equal to 0.05 (5%);

ω_n = natural oscillation period, given by $2\pi/T$;

ω_d = damped oscillation period, given by $\omega_n\sqrt{1-\zeta^2}$.

The elastic response spectra of the acceleration is given through the following steps:

- ❑ The damping factor of the oscillator (normally 5%) and the interval of periods (usually 0-4 seconds) are imposed.
- ❑ ω_n e ω_d are calculated for each value of T.
- ❑ For each value of t, t varying from 0 (start of the recorded accelerogram) till t_{sisma} (end of the recording), the Duhamel's integral is solved.
- ❑ The numerical solution gives n displacements y for every period T; drawing the maximum calculated value of y (y_{max}) as a function of T, the elastic response spectrum of the acceleration is obtained.
- ❑ Starting from the spectrum of the acceleration, it's possible to estimate the spectra of velocity and displacement (PSV and PSD), simply multiplying the graphical ordinates by ω_n e ω_n^2.

It's important to notice that the link between accelerogram and spectra is not biunivocal: different accelerograms could give similar spectra.

Elastic response spectra allow to estimate, in a fast and simply way, the seismic stress acting on the building, owning a natural period of oscillation T>0, during an earthquake (for T=0 the spectrum gives the acceleration of the ground).

Elastic response spectra after Eurocode 8.

Eurocode 8 introduces an elastic response spectrum of acceleration, relative to the horizontal component of the motion, having the following form:

$$0 \le T \le T_B : S_e(T) = a_g \cdot S \cdot \left[1 + \frac{T}{T_B} \cdot (\eta \cdot 2{,}5 - 1)\right]$$

$$T_B \le T \le T_C : S_e(T) = a_g \cdot S \cdot \eta \cdot 2{,}5$$

$$T_C \le T \le T_D : S_e(T) = a_g \cdot S \cdot \eta \cdot 2{,}5\left[\frac{T_C}{T}\right]$$

$$T_D \le T \le 4\text{s} : S_e(T) = a_g \cdot S \cdot \eta \cdot 2{,}5\left[\frac{T_C T_D}{T^2}\right]$$

where the parameter a_g is the horizontal peak ground acceleration at the bedrock level, expressed in g, while the periods TB, TC e TD are given by the following tables, as a function of the surface waves magnitude (M_S) taken as reference:

Ground type	S	T_B (s)	T_C (s)	T_D (s)
A	1,0	0,15	0,4	2,0
B	1,2	0,15	0,5	2,0
C	1,15	0,20	0,6	2,0
D	1,35	0,20	0,8	2,0
E	1,4	0,15	0,5	2,0

Type 1 (M_S>5.5)

Ground type	S	T_B (s)	T_C (s)	T_D (s)
A	1,0	0,05	0,25	1,2
B	1,35	0,05	0,25	1,2
C	1,5	0,10	0,25	1,2
D	1,8	0,10	0,30	1,2
E	1,6	0,05	0,25	1,2

Type 2 ($M_S \leq 5.5$)

The parameter η is the damping correction factor given by

$$\eta = \left[\frac{10}{5+\xi}\right]^{0.5} \geq 0.55.$$

where ζ is the viscous damping, usually equal to 5%, thus η =1.
S is the stratigraphic factor of amplification, which could be possibly multiplied by St, the topographic amplification factor.

The elastic response spectra of acceleration relative to the vertical component of the motion is described by the following relations:

$$0 \leq T \leq T_B : S_{ve}(T) = a_{vg} \cdot \left[1 + \frac{T}{T_B} \cdot (\eta \cdot 3{,}0 - 1)\right]$$

$$T_B \leq T \leq T_C : S_{ve}(T) = a_{vg} \cdot \eta \cdot 3{,}0$$

$$T_C \leq T \leq T_D : S_{ve}(T) = a_{vg} \cdot \eta \cdot 3{,}0 \left[\frac{T_C}{T}\right]$$

$$T_D \leq T \leq 4\text{s} : S_{ve}(T) = a_{vg} \cdot \eta \cdot 3{,}0 \left[\frac{T_C . T_D}{T^2}\right]$$

where the parameter a_{vg} is the vertical peak ground acceleration at the bedrock level, expressed in g, while the periods TB, TC e TD are given by the following tables, as a function of the surface waves magnitude (M_S) taken as reference:

Spectrum	a_{vg}/a_g	T_B (s)	T_C (s)	T_D (s)
Type 1	0,90	0,05	0,15	1,0
Type 2	0,45	0,05	0,15	1,0

Type 1 (M_S>5.5) - Type 2 ($M_S \leq 5.5$)

Elastic response spectra by the application of the ground motion prediction equations (GMPEs): Boore e Atkinson (2008).

GMPEs (Ground Motion Prediction Equations) are empirical correlations which link a specific seismic parameter, usually PGA, PGV or the spectral accelerations, to the magnitude and to the epicentral, ipocentral or seismic fault distance.
The relation by Boore and Atkinson is based on the following equation:

$$\ln Y = F_M(\mathbf{M}) + F_D(R_{JB}, \mathbf{M}) + F_S(V_{S30}, R_{JB}, \mathbf{M}) + \varepsilon\sigma_T ,$$

where the funcions F_M, F_D e F_S represent, respectively, the magnitude scaling, the distance function and the site amplification.
M is the moment magnitude of the seism, R_{JB} is the Joiner-Boore distance, defined as the closest distance to the seismic fault projection on the ground surface (approximately equal to the epicentral distance when M<6) and V_{s30} is the weighted average of the shear wave velocities to a depth of up to 30 m. The parameter ε represents the distance, expressed ad standard deviation, of a single predicted value of lnY from the mean value of lnY; σ_T is function of the aleatory uncertainty of the sperimental data.

The function F_M is given by:

(a) $\mathbf{M} \leq \mathbf{M}_h$

$$F_M(\mathbf{M}) = e_1 U + e_2 SS + e_3 NS + e_4 RS + e_5(\mathbf{M} - \mathbf{M}_h) + e_6(\mathbf{M} - \mathbf{M}_h)^2$$

(b) $\mathbf{M} > \mathbf{M}_h$

$$F_M(\mathbf{M}) = e_1 U + e_2 SS + e_3 NS + e_4 RS + e_7(\mathbf{M} - \mathbf{M}_h)$$

where:
M_h = hinge magnitude, parameter as a function of the spectral period T;
e_1, e_2, e_3, e_4, e_5, e_6, e_7 = coefficients as a function of the spectral period T;
U, SS, NS, RS = variables having the values 0 or 1 as a function of the fault type (U=unspecified, SS=strike-slip, NS=normal, RS=thrust/reverse) based on the following scheme:

Fault Type	U	SS	NS	RS
unspecified	1	0	0	0
strike-slip	0	1	0	0
normal	0	0	1	0
thrust/reverse	0	0	0	1

The function F_D has the following expression:

$$F_D(R_{JB}, \mathbf{M}) = [c_1 + c_2(\mathbf{M} - \mathbf{M}_{ref})] \ln(R / R_{ref}) + c_3(R - R_{ref})$$

where:

$$R = \sqrt{R_{JB}^2 + h^2}$$

being h, c_1, c_2, c_3, M_{ref} a R_{ref} parameters as a function of the spectral period T.
M_{ref} = 4.5;
R_{ref} = 1.0.

Finally, the function F_S is given through the calculation procedure seen in a previous paragraph (seismic amplification after Boore e Atkinson).
Below tables of the parameters as a function of the spectral period are displayed.

period	b_{lin}	b_1	b_2
PGV	-0.600	-0.500	-0.06
PGA	-0.360	-0.640	-0.14
0.010	-0.360	-0.640	-0.14
0.020	-0.340	-0.630	-0.12
0.030	-0.330	-0.620	-0.11
0.050	-0.290	-0.640	-0.11
0.075	-0.230	-0.640	-0.11
0.100	-0.250	-0.600	-0.13
0.150	-0.280	-0.530	-0.18
0.200	-0.310	-0.520	-0.19
0.250	-0.390	-0.520	-0.16
0.300	-0.440	-0.520	-0.14
0.400	-0.500	-0.510	-0.10
0.500	-0.600	-0.500	-0.06
0.750	-0.690	-0.470	0.00
1.000	-0.700	-0.440	0.00
1.500	-0.720	-0.400	0.00
2.000	-0.730	-0.380	0.00
3.000	-0.740	-0.340	0.00
4.000	-0.750	-0.310	0.00
5.000	-0.750	-0.291	0.00
7.500	-0.692	-0.247	0.00
10.000	-0.650	-0.215	0.00

period	c_1	c_2	c_3	h
pga4nl	-0.55000	0.00000	-0.01151	3.00
PGV	-0.87370	0.10060	-0.00334	2.54
PGA	-0.66050	0.11970	-0.01151	1.35
0.010	-0.66220	0.12000	-0.01151	1.35
0.020	-0.66600	0.12280	-0.01151	1.35
0.030	-0.69010	0.12830	-0.01151	1.35
0.050	-0.71700	0.13170	-0.01151	1.35
0.075	-0.72050	0.12370	-0.01151	1.55
0.100	-0.70810	0.11170	-0.01151	1.68
0.150	-0.69610	0.09884	-0.01113	1.86
0.200	-0.58300	0.04273	-0.00952	1.98
0.250	-0.57260	0.02977	-0.00837	2.07
0.300	-0.55430	0.01955	-0.00750	2.14
0.400	-0.64430	0.04394	-0.00626	2.24
0.500	-0.69140	0.06080	-0.00540	2.32
0.750	-0.74080	0.07518	-0.00409	2.46
1.000	-0.81830	0.10270	-0.00334	2.54
1.500	-0.83030	0.09793	-0.00255	2.66
2.000	-0.82850	0.09432	-0.00217	2.73
3.000	-0.78440	0.07282	-0.00191	2.83
4.000	-0.68540	0.03758	-0.00191	2.89
5.000	-0.50960	-0.02391	-0.00191	2.93
7.500	-0.37240	-0.06568	-0.00191	3.00
10.000	-0.09824	-0.13800	-0.00191	3.04

Period	e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\mathbf{M}_h$
pga4nl	-0.03279	-0.03279	-0.03279	-0.03279	0.29795	-0.20341	0.00000	7.00
PGV	5.00121	5.04727	4.63188	5.08210	0.18322	-0.12736	0.00000	8.50
PGA	-0.53804	-0.50350	-0.75472	-0.50970	0.28805	-0.10164	0.00000	6.75
0.010	-0.52883	-0.49429	-0.74551	-0.49966	0.28897	-0.10019	0.00000	6.75
0.020	-0.52192	-0.48508	-0.73906	-0.48895	0.25144	-0.11006	0.00000	6.75
0.030	-0.45285	-0.41831	-0.66722	-0.42229	0.17976	-0.12858	0.00000	6.75
0.050	-0.28476	-0.25022	-0.48462	-0.26092	0.06369	-0.15752	0.00000	6.75
0.075	0.00767	0.04912	-0.20578	0.02706	0.01170	-0.17051	0.00000	6.75
0.100	0.20109	0.23102	0.03058	0.22193	0.04697	-0.15948	0.00000	6.75
0.150	0.46128	0.48661	0.30185	0.49328	0.17990	-0.14539	0.00000	6.75
0.200	0.57180	0.59253	0.40860	0.61472	0.52729	-0.12964	0.00102	6.75
0.250	0.51884	0.53496	0.33880	0.57747	0.60880	-0.13843	0.08607	6.75
0.300	0.43825	0.44516	0.25356	0.51990	0.64472	-0.15694	0.10601	6.75
0.400	0.39220	0.40602	0.21398	0.46080	0.78610	-0.07843	0.02262	6.75
0.500	0.18957	0.19878	0.00967	0.26337	0.76837	-0.09054	0.00000	6.75
0.750	-0.21338	-0.19496	-0.49176	-0.10813	0.75179	-0.14053	0.10302	6.75
1.000	-0.46896	-0.43443	-0.78465	-0.39330	0.67880	-0.18257	0.05393	6.75
1.500	-0.86271	-0.79593	-1.20902	-0.88085	0.70689	-0.25950	0.19082	6.75
2.000	-1.22652	-1.15514	-1.57697	-1.27669	0.77989	-0.29657	0.29888	6.75
3.000	-1.82979	-1.74690	-2.22584	-1.91814	0.77966	-0.45384	0.67466	6.75
4.000	-2.24656	-2.15906	-2.58228	-2.38168	1.24961	-0.35874	0.79508	6.75
5.000	-1.28408	-1.21270	-1.50904	-1.41093	0.14271	-0.39006	0.00000	8.50
7.500	-1.43145	-1.31632	-1.81022	-1.59217	0.52407	-0.37578	0.00000	8.50
10.000	-2.15446	-2.16137	0.00000	-2.14635	0.40387	-0.48492	0.00000	8.50

Elastic response spectrum adapted to the Eurocode 8 types through the ICMS 2008 procedure.

Elastic response spectra, generated by an accelerogram, have an irregular shape. To simplify the using, the spectrum is usually modified through common techniques of smoothing (for example simple or exponential moving average).

It's possible to generate a regular spectrum, adapted to the Eurocode 8 types, using as a reference the coordinates of the maximum amplitude of the calculated spectrum (Tmax, PSAmax), employing the following parameters:

S=Fa;
St=1;
Tc=2π(Fv/Fa);
Tb=Tc/3;

Td=2.0 or 1.2, as a function of the spectrum type (type 1 or type 2)

The parameter η is the damping correction factor

$$\eta = \left[\frac{10}{5+\xi}\right]^{0.5}$$

where ζ is the viscous damping, usually equal to 5%, thus η =1.
Fa and Fv (respectively the spectral amplification of acceleration and velocity) are evaluated through a comparison between the response spectra calculated at the ground surface and at the bedrock.
The steps to calculate Fa are the following:

- the period T, corresponding to the maximum value of acceleration in the spectrum of the bedrock, is identified (T_{max});
- the mean value of the ordinate around T_{max} is estimated by the following relation:

$$A_{medio} = \frac{1}{T_{\max}} \int_{0,5T_{\max}}^{1,5T_{\max}} A(T)dT$$

- the steps 1 and 2 are repeated for the surface spectrum of the acceleration;
- Fa is calculated as the ratio between A_{medio} at the ground surface and at the bedrock:

$$Fa = \frac{A_{medio\sup erficie}}{A_{mediobedrock}}$$

The steps to calculate Fv are the following:

- the period T, corresponding to the maximum value of velocity in the spectrum of the bedrock, is identified (T_{max});
- the mean value of the ordinate around T_{max} is estimated by the following relation:

$$V_{medio} = \frac{1}{T_{\max}} \int_{0,8T_{\max}}^{1,2T_{\max}} V(T)dT$$

- the steps 1 and 2 are repeated for the surface spectrum of the velocity;
- Fv is calculated as the ratio between V_{medio} at the ground surface and at the bedrock:

$$Fv = \frac{V_{medio\,\sup erficie}}{V_{mediobedrock}}$$

Elastic response spectra of velocity and displacement.

Starting from the response spectrum of acceleration, elastic response spectra of velocity and displacement can be drawn, approximately, multiplying the spectral amplitudes by the factor T/2π, where T is the spectral period:

- velocity spectrum:

$$PSV = \frac{T}{2\pi} A(T)$$

- displacement spectrum:

$$PSD = \left(\frac{T}{2\pi}\right)^2 A(T)$$

Housner Intensity

The Housner Intensity is a parameter associated to the elastic response spectrum of velocity. It's defined as the integral of PSV, for a viscous damping set to 5%, extended to a specific interval of T (spectral period), usually between 0.1 e 2.5 s.

$$I_H = \int_{T1}^{T2} PSV(T)dT$$

Seismic geotechnical effects.

Seismic generated pore pressure in granular soil.

Under specific conditions, an earthquake can produce an increment of the pore pressure Δu for the strain inside a granular soil due to the passage of the seismic waves.
In case of granular soil the raising of Δu can be estimated through the relation of Seed e Booker (1977), which links Δu to number of load cycles N generated by the seism:

$$\Delta u_N = \sigma'_0 \frac{2}{\pi} sen^{-1}\left[\left(\frac{N}{N_L}\right)^{0.5a}\right]$$

where N_L is the number of load cycles necessary to produce liquefaction in the saturated soil layer, σ'_0 is the effective mean pressure in static condition and *a* is a factor associated to the relative density, expressed in decimal format, through the relation (Fardis e Veneziano, 1981):

$$a=0.96D_r^{0.83}$$

the parameter N_L can be, approximately, given by the following table (Seed et al., 1975):

N_L	τ_{medio}/σ'_0
100	0.09
30	0.12
10	0.15
3	0.22

where τ_{medio} is the mean shear stress due to the seism, usually sets equal to 65% of τ_{max}.
N can be correlated to the magnitude through the following table (Seed et al., 1975):

N	M
3.8	5
4.0	5.5
4.5	6.0
7.0	6.5
10.0	7

Once Δu is estimated, the angle of shear resistence in seismic condition is given by:

$$\tan\varphi^* = \left(1 - \frac{\Delta u}{\sigma_0'}\right)\tan\varphi$$

Seismic generated pore pressure in cohesive soil.

A simply relation, which permits to estimate approximately the increment of the pore pressure Δu in seismic condition inside a cohesive soil, is given by Matsui et al. (1980):

$$\Delta u = \sigma'_0\ \beta Log_{10}\left(\frac{\gamma_{max}}{\gamma_s}\right)$$

where:

σ'_0 = effective mean pressure given by:

$$\sigma'_0 = \sigma'_{v0}\frac{1+2k_0}{3}$$

where σ'_{v0} is the effective vertical pressure and k_0 is the at rest stress ratio, calculable through the formula:

$$k_0 = 1 - sen\varphi$$

being φ the angle of shear resistance of the soil layer;

β = empirical factor equal to 0.45;

γ_s = strain associated to the volumetric threshold, estimable by the formula:

$$\gamma_s = A(OCR-1)+B$$

where A and B are given as a function of the plasticity index IP:
IP= 20% - A=0.4 10^{-3} B=0.6 10^{-3}
IP= 40% - A=1.2 10^{-3} B=1.1 10^{-3}
IP= 55% - A=2.5 10^{-3} B=1.2 10^{-3}

γ_{max} = maximum soil strain due to the seism, given by:

$$\gamma_{\max} = \frac{\tau_{\max}}{G_\gamma}$$

where τ_{max} is the maximum shear stress produced by the eathquake:

$$\tau_{\max} = a_g \sigma_v r_d$$

with a_g the seismic acceleration (g), σ_v the total vertical pressure and r_d = 1-0.015Z, where Z is the depth to the ground surface; G_γ is the shear modulus associated to the strain γ; G_γ , for high strain, is always lesser than G_0, modulus associated to low strain, and, in case of γ values close to the volumetric threshold, it can be setted approximately close to 0.75 G_0; it reminds G_0 is correlable to the S wave velocity by the analytical expression:

$$G_0 = \rho V_s^2$$

where ρ is the mass density of the soil layer given by the ratio between the soil unit weight and the acceleration of gravity (9,81 m/s^2).

The Matsui relation involves that, to have a positive raising of the pore pressure, γ_{max} must be higher than γ_s. This means that important increments of Δu can be reached only in case of high seismic forces in soil layers having low G0 values.

In a cohesive soil with plasticity index less than or equal to 55%(IP≤55%), in undrained condition it can be observed a decreasing of cu (undrained cohesion) due to the load cycles caused by the seism. The degradation index can be estimated, in case where $\Delta u / \sigma'_{v0} > 0.5$, by the following relation:

$$\delta c_u = \sqrt{\frac{c_u - \Delta u}{c_u}}$$

where instead it's $\Delta u / \sigma'_{v0} \leq 0.5$ δc_u can be calculated by:

$$\delta c_u = N^{-t}$$

where N is the number of load cycles produced by the earthquake (see above) and t is a parameter function of the cyclic shear strain associated to the seism $\gamma_c = \frac{\tau_{media}}{G_\gamma}$ and of the strain linked to the volumetric threshold(see above):

$$t = s(\gamma_c - \gamma_s)^r$$

The parameters s and r are given by the following table:(Matasovic, 1993):

	OCR=1			OCR=2	OCR=4
	IP=15%	IP=30%	IP=50%	IP=50%	IP=50%
s	0.195	0.095	0.075	0.054	0.042
r	0.600	0.600	0.495	0.480	0.423

Finally the corrected undrained cohesion is calculable by the following formula:

$$c_{uc} = \delta c_u x c_u$$

Once Δu is estimated, the post-seismic settlement of the soil layer is given by the following relation:

$$\Delta H = \frac{\alpha C_r}{1+e_0} \log\left[\frac{1}{1-\frac{\Delta u}{\sigma'_0}}\right] H$$

where:

H = layer thickness;

C_r = recompression index;

e_0 = void ratio;

α = empirical costant, usually setted equal to 1.

Dynamic stability analysis of an infinite slope by the Newmark method.

Simplified dynamic analysis of a slope allows to calculate the cumulative displacements produced by an earthquake, through the processing of its accelerogram, inside a potentially unstable slope. The displacement method, originally developed by Newmark (1965) involves they are satisfied the following conditions:

- the reference accelerogram be valid for the whole examined slope;
- the shear resistance of the soil layers be the same both in static and dynamic conditions;
- the potential landslide could not move upward.

The calculation steps are described below.

1)Calculate the critical horizontal seismic acceleration k_c of the slope, that's the a_g value for which is F_s=1. In case of infinite slope k_c has the following form:

$$k_c = \frac{\frac{cb}{\cos\alpha} + \left[(W + Qb)\cos\alpha - U\right]\tan\varphi - (W + Qb)sen\alpha}{\frac{1}{2}W\cos\alpha}$$

2)Examine the recorded $a_g(t)$ values by the accelerogram using a reading step Δt equal to the recording one; the landslide motion starts at time t_0 for which is $a_g(t) \geq k_c$.

3)Calculate the landslide displacement through a double numerical integration of $a_r(t)=a_g(t)-k_c$, to get the following relations:

$$(1)\, s(t+\Delta t) = s(t) + v(t)\Delta t + \frac{2a_r(t) + a_r(t+\Delta t)}{6}\Delta t^2$$

$$(2)\, v(t+\Delta t) = v(t) + \frac{a_r(t) + a_r(t+\Delta t)}{2}\Delta t\ ;$$

Naturally at time t=0 they are s(t)=0 and v(t)=0.

4)Multiply the calculated displacement, relative to a rigid block lying on a horizontal sliding plane, by a factor A as a function of the real shape of the sliding plane. In case of a planar surface, A has the following form:

$$A = \frac{\cos(\varphi - \alpha)}{\cos\varphi}$$

where φ is the angle of shear resistance acting along the sliding plane and α is the slope of the plane itself.

5)Apply the formula (1) and (2) to the next Δt intervals, through an iterative process until it will be verified the following condition:

$$v(t + \Delta t) = v(t) + \frac{a_r(t) + a_r(t + \Delta t)}{2}\Delta t = 0$$

6)Proceed reading the ag(t) values till the end of recording, repeating the steps 2) and 3) till when the condition ag(t)≥kc is verified.

The cumulative displacement in the intervals where $a_g(t) \geq a_c$ can be used to have an estimation of the damage level caused by the seism to the structures lying on the slope based on the following table (Legg e Slosson, 1984):

Damage level	Cumulative displacement (cm)
Irrelevant	<0.5
Moderate	0.5-5
Strong	5-50
Severe	50-500
Catastrophic	>500

As to the maximum tollerable displacement, that's the displacement beyond which the slope has to be reguarded as unstable, it can refer to the folowing scheme(ASCE, 2002 e Wilson e Keefer, 1985):

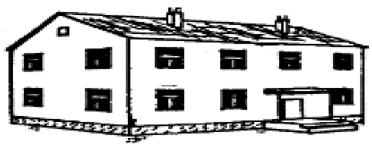
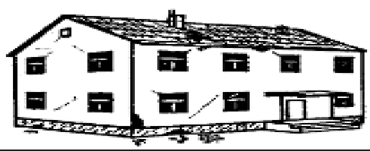
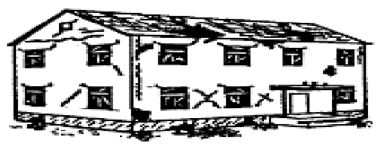
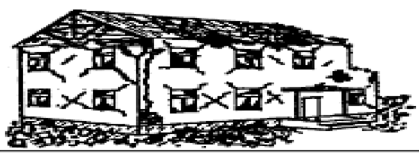
Description	Max.displacement (cm)
Rock slope	2
Slope with engineering structures	5
Slope without engineering structures having a ductile behaviour (loose sand and gravel, N.C. clay)	15
Slope without engineering structures having a not ductile behaviour (dense sand and gravel, S.C. clay) where the peak shear strength is mobilised.	5
Slope without engineering structures having a not ductile behaviour (dense sand and gravel, S.C. clay) where the residual or ultimate shear strength is mobilised.	15

Empirical assessment of the seismic hazard of a building.

A first evaluation of the seismic hazard of single building can be performed, using Fragility Curves. They are charts, distinct on the basis of building typology, where on the X axis a parameter associated to the seism (PGA, Intensità di Housner, ect.) is displayed, while on the Y axis the probability of not exceedance of a specific damage level are marked.

Fragility Curves: Rota et al. (2008)

For the building typology identified inside the european territory, a set of fragility curves has been processed by Rota et al.(2008). They are expressed as a function of the Housner Intensity and are linked to the European Macroseismic Scale (EMS 98) of damage.

Classification of damage to masonry buildings	
	**Grade 1: Negligible to slight damage (no structural damage, slight non-structural damage)** Hair-line cracks in very few walls. Fall of small pieces of plaster only. Fall of loose stones from upper parts of buildings in very few cases.
	**Grade 2: Moderate damage (slight structural damage, moderate non-structural damage)** Cracks in many walls. Fall of fairly large pieces of plaster. Partial collapse of chimneys.
	**Grade 3: Substantial to heavy damage (moderate structural damage, heavy non-structural damage)** Large and extensive cracks in most walls. Roof tiles detach. Chimneys fracture at the roof line; failure of individual non-structural elements (partitions, gable walls).
	**Grade 4: Very heavy damage (heavy structural damage, very heavy non-structural damage)** Serious failure of walls; partial structural failure of roofs and floors.
	**Grade 5: Destruction (very heavy structural damage)** Total or near total collapse.

Curves have been processed as a function of the building typology and the number of storeys, supposing a lognormal distribution of the probability.

Label	Building class	No. of storeys
MX1	Mixed	1-2
MX2	Mixed	≥3
RC1	Reinforced concrete – seismic design	1-3
RC2	Reinforced concrete – no seismic design	1-3
RC3	Reinforced concrete – seismic design	≥4
RC4	Reinforced concrete – no seismic design	≥4
IMA1	Masonry – irregular layout – flexible floors – with tie rods and/or tie beams	1-2
IMA2	Masonry – irregular layout – flexible floors– w/o tie rods and tie beams	1-2
IMA3	Masonry – irregular layout – rigid floors – with tie rods and/or tie beams	1-2
IMA4	Masonry – irregular layout – rigid floors - w/o tie rods and tie beams	1-2
IMA5	Masonry – irregular layout – flexible floors – with tie rods and/or tie beams	≥3
IMA6	Masonry – irregular layout – flexible floors– w/o tie rods and tie beams	≥3
IMA7	Masonry – irregular layout – rigid floors – with tie rods and/or tie beams	≥3
IMA8	Masonry – irregular layout – rigid floors - w/o tie rods and tie beams	≥3
RMA1	Masonry – regular layout – flexible floors – with tie rods and/or tie beams	1-2
RMA2	Masonry – regular layout – flexible floors – w/o tie rods and tie beams	1-2
RMA3	Masonry – regular layout – rigid floors – with tie rods and/or tie beams	1-2
RMA4	Masonry – regular layout – rigid floors – w/o tie rods and tie beams	1-2
RMA5	Masonry – regular layout – flexible floors – with tie rods and/or tie beams	≥3
RMA6	Masonry – regular layout – flexible floors – w/o tie rods and tie beams	≥3
RMA7	Masonry – regular layout – rigid floors – with tie rods and/or tie beams	≥3
RMA8	Masonry – regular layout – rigid floors – w/o tie rods and tie beams	≥3
ST	Steel	All

(Rota et.al. 2003)

The mean value (μ) and the standard deviation(σ) of every building typology is tabled in the following scheme:

Label	DS1		DS2		DS3		DS4		DS5	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
MX1	1.60	4.36	7.69	5.88	13.64	8.95	11.16	5.01	9.13	2.90
MX2	1.20	5.67	6.83	4.44	6.97	3.18	9.70	4.01	8.91	2.58
RC2	5.30	3.80	11.04	4.83	9.26	2.85	7.84	1.85	-	-
RC4	4.25	2.91	7.90	3.11	7.84	2.55	8.28	2.06	-	-
IMA1	-9.94	13.23	10.23	17.01	12.15	10.20	10.89	5.82	9.41	3.03
IMA2	-7.97	9.30	2.31	13.80	8.23	15.27	11.57	10.64	11.27	6.36
IMA3	-1.98	13.92	11.99	11.25	14.12	9.14	13.41	6.21	-	-
IMA4	-7.18	11.33	4.27	6.26	6.15	5.11	6.89	3.43	6.99	2.34
IMA5	-8.92	10.36	3.10	5.94	4.79	2.92	13.65	7.85	10.54	3.91
IMA6	-7.28	8.11	2.37	10.65	5.43	6.48	7.30	4.63	8.98	3.93
IMA7	0.93	4.15	3.58	1.83	4.56	1.90	7.29	3.08	8.32	2.50
IMA8	-5.84	8.58	2.73	4.23	3.92	2.53	5.27	2.26	6.58	2.12
RMA1	0.89	8.01	15.56	12.26	13.59	7.69	10.39	3.59	6.31	0.95
RMA2	-9.01	14.89	7.42	11.16	11.72	10.91	14.21	8.77	11.85	5.06
RMA3	6.84	14.80	11.51	6.55	11.27	5.12	8.95	2.98	7.19	1.65
RMA4	0.37	16.33	18.50	15.35	16.05	9.75	12.77	5.43	8.53	2.27
RMA5	-0.14	8.81	7.98	9.60	13.20	9.08	9.22	3.36	6.77	1.36
RMA6	-2.39	6.56	3.69	3.46	4.97	3.18	6.23	2.81	9.24	3.69
RMA7	3.47	5.06	10.05	7.07	7.58	3.39	11.50	4.67	8.98	2.61
RMA8	-0.61	11.59	6.99	7.61	7.25	4.47	7.63	2.98	8.60	2.73
MX1	1.60	4.36	7.69	5.88	13.64	8.95	11.16	5.01	9.13	2.90
MX2	1.20	5.67	6.83	4.44	6.97	3.18	9.70	4.01	8.91	2.58

(Rota et.al. 2003)

Fragility curves: Rossetto et al. (2003)

The Fragility curves, processed by Rossetto et al, refer to RC buildings and have the following form:

$$P = 1 - \exp(-\alpha GM^{\beta})$$

where GM is the reference parameter of the seism (PGA, spectral acceleration $S_{a5\%}$ or spectral displacement $S_{d5\%}$ with 5% viscous damping). The parameters α and β are tabled as a function of the selected GM, of the damage level and of the reference percentile.

	Mean		U90%		L90%		
	α	β	α	β	α	β	
							PGA
Slight	1.556	1.60	3.950	1.60	0.830	1.60	339872
Light	1.055	1.80	2.732	1.80	0.620	1.80	339187
Moderate	0.250	3.00	0.903	3.00	0.102	3.00	331702
Extensive	0.093	4.00	0.538	4.00	0.010	4.00	329152
P.Collapse	0.009	5.80	0.162	5.80	0.001	5.80	292839
Collapse	0.001	8.00	0.005	8.00	0.001	8.00	77876
							$Sa_{5\%}$(T$_{ela}$
Slight	0.633	1.80	1.865	1.80	0.192	1.80	339872
Light	0.396	1.80	1.356	1.80	0.116	1.80	339187
Moderate	0.153	1.80	0.524	1.80	0.041	1.80	331702
Extensive	0.090	2.00	0.447	2.00	0.036	2.00	329152
P.Collapse	0.050	2.20	0.265	2.20	0.031	2.20	292839
Collapse	0.010	3.00	0.056	3.00	0.006	3.00	77876
							$Sd_{5\%}$(T$_{ela}$
Slight	25.82	1.10	76.45	1.10	13.72	1.10	339872
Light	21.08	1.20	73.88	1.20	8.350	1.20	339187
Moderate	6.500	1.15	29.57	1.15	2.342	1.15	331702
Extensive	3.000	1.30	17.52	1.30	1.323	1.30	329152
P.Collapse	2.500	2.00	13.45	2.00	1.200	2.00	292839
Collapse	2.000	2.40	9.37	2.40	1.119	2.40	77876

(Rossetto et.al. 2003)