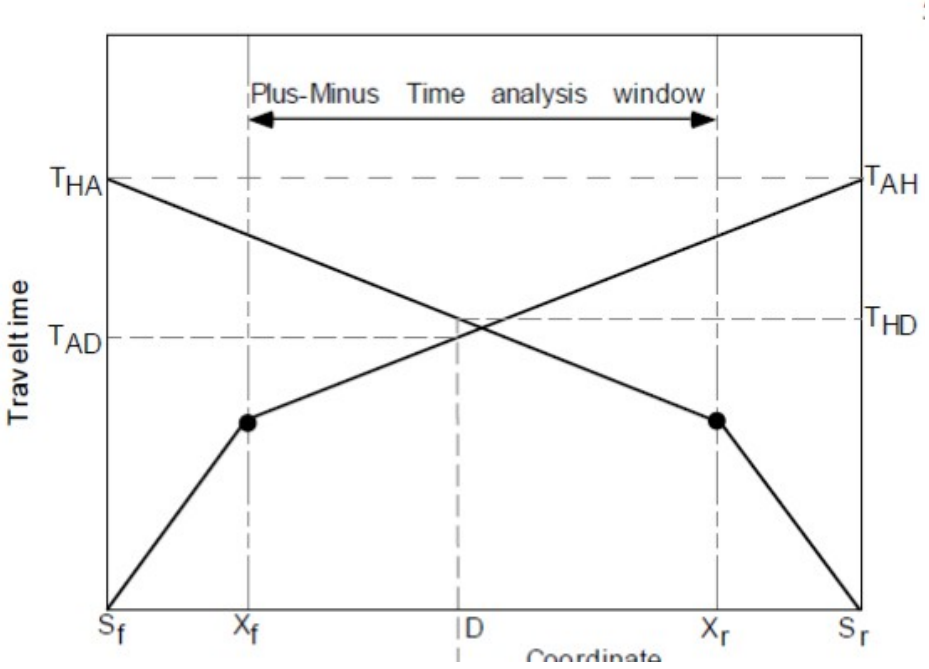
Index

1. Theoretical bases

1.1 Plus-minus method

The Plus-minus method (Hagedoorn, 1959 e van Overmeeran, 1987) is based on the assumption that the arrival times of the P and S waves from the source to the receiver be identich if their positions are reversal.
On the hypotesis of a two-layer ground with two sources (Sf and Sr) placed at the external limits of the array, the analysis window is individuated by the points Xf and Xt (see picture), defining the dromochone points from which the refracted waves of the second layer arrive to the receivers, respectivley from the sources Sf and Sr.



Supposing to have a receiver in position D, Plus time at the receiver D (T^{+}_{D}) is given by:

$$T^{+}D = T_{AD} + T_{HD} - T_{AH}$$

where T_{AD} is the travel time of the signal from the source Sf to D, T_{HD} is the travel time from source Sr and D and T_{AH} is the travel time from Sf to Sr, the later gotten by extrapolating the dromochrone of the second layer up to reach X of the two sources.
Plus time allows to assess the bottom depth of the first layer at position D by the relationship:

$$Z_1D = [(T^{+}D)*(V_1)]/2(\cos(\theta_C))$$

where:

$$\theta_C = \sin^{-1}(V_1/V_2)$$

and V_1 is the velocity of the first layer (direct wave), given by the inverse slope of the first tract of the dromochone (from Sf to Xf and from Sr to Xr).
V_2 is the second-layer velocity, given by Minus time at the receiver D (T^{-}_{D}) :

$$T^{-}D = T_{AD} - T_{HD} - T_{AH}$$

Considering a second receiver D' placed at a distance ΔX from D, with a Minus time $T^{-}_{D'}$, one gets

$$V_2 = 2(\Delta X)/\Delta T^{-}D$$

where ΔT^{-}_{D} is the difference between T^{-}_{D} and $T^{-}_{D'}$.
Naturally the calculation can be extended to a whatever number of layers.
Plus-minus methiod has the following limits:

1. the refractor morphology may have an irregular profile, however the single tracts cannot have an excessive slope (generally not exceeding 20°);

2. One cannot individue lateral variation of velocities inside the same refractor.

1.2 Refraction tomography

Processing of a tomographic model by refraction seismic can be divided in two different steps.

Forward problem

In the first step the soil ground is subdivided in a mesh of nodes, sufficiently dense. At each node is assigned an initial value of slowness, inverse of velocity (1/V), S, given by the starting model computed, e.g., by the Plus-minus method. Based on this mesh, the path of the first arrivals from the sources to the receivers (ray tracing) is simulated, using the Dijkstra's algorithm, improved by a Fibonacci heap, to get the shortest paths.
Then arrival time of the ray from the source to the receiver is given by:

$$t = \sum_{i=1}^{M} l_i s_i$$

where M is the number of nodes crossed by the ray, l is the length of the tract between a node and the next one and s is the average slowness of the tract.

Inverse problem

After obtaining the shortest paths from sources to receivers, the computed arrival times are confronted with the measured ones. Discrepancies are distributed along the nodes of the single paths, using techniques as the Algebraic Reconstruction Techniques (ART). Calculation is repeated till to reach a neglectable discrepancies.