PROGRAM GEO – Soils ver.3

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1. Theoretical basis

1.1 Problem definition.

Procedures to analyse soil slope stability, through assesment of the limit equilibrium, consist of estimating of a safety factor relative to translational and/or rotational equilibriumof the soil volume between the ground profile and the potential slip surface imposed.
Calculation procedure takes in account the whole set of forces and moments working along a shear plane, giving an assessment of the global stability through the equilibrium equations.
A global safety factor is calculated by the ratio between the maximum available shear resistance along the collapsing surface and the mobilized strength along the same surface:

$$Fsic = Tmax / Tmob;$$

with
Fsic= safety factor;
Tmax= available shear resistance;
Tmob= mobilized strength.

At the equilibrium (Tmax=Tmob) Fsic has to be equal to 1.

1.2 Setting the calculation procedure.

To apply the static equations to the soil slope analysis, the following conditions have to be verified:
a) verify has to be executed taking in account a section of slope having unitary width (usually 1 m), negleting lateral interactions between this slice and the surrounding ground;
b) shear resistance along the potential collapsing surface has to be expressibled by the Mohr-Coulomb law:

$$Tmax = c + \gamma\ h\ tg\ \varphi;$$

with
Tmax = maximum shear resistance of the soil;
c= cohesion of the soil;
γ= unit weight of the soil;
h= depth of the collapsing slip;
φ = angle of shearing resistance of the soil.

c)accuracy with which the geotechnical parameters are assessed, in situ or in laboratory, has to be the same: otherwise the mobilized shear resistance has to be expressed by the following way:

$$Tmob = (c/Fsicc) + (\gamma\ h\ tg\ \varphi/Fsicp);$$

with
Fsiic =safety factor related to c;
Fsicp = safety factor related to φ;

d)there has to be an homogeneous distribution of the tangent stesses mobilized (Tmob) along the potential collapsing surface, that is in every point along the hypotetical slip plane the parameters of the Mohr-Coulomb equation, c, φ, γ and h, have to have the same values.
To limit the error inserted in the calculation by this last hypotesis, the slip surface, in the most part of the procedures noted in letterature, is divided in

more slices, inside of which the homogeneous distribution of Tmob is considered to be satisfied.
By a practical point of view, slices are placed where either there is a variation of the geotechnical parameters or there is a significant changes in the topographic profile. This way to set the problem drives however to the introduction in the analytical solution of new unknown variables, related to the way in which slices interacting each other along the contact lines.
Consequently, in the calculation of the safety factor they take part the following unknown variables (n=number of slices to be considered):

a) the normal forces (N) working on slices (n unknown variables);

b) the tangential forces (T) working on slices (n unknown variables);
c) the points of application of the normal and tangential forces on slices (n unknown variables);
d) the horizontal forces acting on the dividing surfaces between contiguous slices (n-1 unknown variables);
e) the vertical forces acting on the dividing surfaces between contiguous slices (n-1 unknown variables);
f) the points of application of the forces d) and e) (n-1 unknown variables);
g) teh safety factor Fsic (1 unknown variable).

Then solution involves an overall set of 6n-2 unknown variables. Actually they are available:

a) 3n equilibrium equations;
b) n equations of this sort:

$$T = (c\ l + N\ tg\ \varphi)/Fsic;$$

with
l = length of the slice;

which link, for each slice, the unknown variables N,T and Fsic.

c) n equations given imposing the points of application of N, T on the central point of the slice base.

In all, 5n equations are available to analytically solve the problem. To get Fsic, there obviously would need as many equation as the unknown variables. So that the problem be statically determined, and consequently solvable, they actually miss still n-2 equations. These equations can be gotten inserting in the analysis some simplifying hypothesis, generally regarding the distribution of forces along the dividing surfaces of contiguous slices.
The several available solving procedures differ each other essentially for the chosen simplifying hypothesis about the force distribution.

1.3 Solving by limit equilibrium methods

1.3.1 Fellenius.

By the Fellenius method the condition that the forces working along the dividing surfaces of contiguous slices are negligible is imposed.
It is a method based on the equilibrium of the moments.
Be:

$$Ni=Wconcio(i) \cos \alpha i;$$

with
Wconcio(i)= weight of the i-th slice;
αi=inclination of the i-th base slice ;
Ni=normal component to the slice base of Wconcio(i).

Imposing the equilibrium of the moments at the centre of the circular potential slip surface of the soil slope, one can write:

$$\Sigma R \text{ sen } \alpha i \text{ Wconcio(i)}=\Sigma R \text{ Ti};$$

where R sen αi is the lever arm of Wconcio(i).
Finally:

$$SF = \Sigma(Ci \text{ Lconcio(i)}+Ni \text{ tg } \varphi i) / \Sigma \text{sen } \alpha i \text{ Wconcio(i)};$$

with
Ci=cohesion working along the slice base i;
Lconcio(i)=lenght of the slice base i;
φi=angle of shearing resistance along the slice base i;

Inserting the contribution due to the water table, one gets:

$$Fsic = \Sigma Ci \text{ Lconcio(i)} + (Ni\text{-hfalda(i) Lconcio(i))tg } \varphi i] / \Sigma \text{Wconcio(i) sen } \alpha i \;;$$

where:
hfalda(i)=height of the piezometric line respect to the slice base i;

This method generally drives to Fsic underestimating respect to more complete methods, particularly in case of cohesive or overconsolidated soils and of depth slip-surfaces.
Error is however on safety side, even if, in some cases, it can overtake 20% respect to more rigorous methods.
It can used both with circular and poligonal slip-surfaces.
This method, like the next ones taken in examination, might sometimes give negative safety factors. It occours when inclination the slip surface gets very high negative values, as in case of very deep slip surface with respect to its lenght. These surfaces are to be considered definitly stableand the calculated SF meaningless.

1.3.2 Bishop (simplified).

By the Bishop method the condition that the vertical forces working along the dividing surfaces of contiguous slices are negligible is imposed. Consequently slices interact among them through horizontal forces only.
It is a method based on the equilibrium of the moments.
One supposes the potential slip surface be circular.
The maximum shearing resistance available along the potential sliding surface is given, for each slice, by

$$Ti\ max = Xi / (1 + Yi / SF);$$

with $Xi = (c + (g \times h - gw \times hw) \times tg\ \varphi) \times dx / \cos \alpha$
where gw = water unit weight;
hw = height of the water table with respect to the slice bottom;
dx = horizontal length of the slice;
α = slice inclination with respect to horizontal plane.

$Yi = tg\ \alpha \times tg\ \varphi$

The mobilized shearing strenght along the shearing plane is, for each slice, given by:

$$Ti\ mob = Zi$$

with Zi = g x h x dx x sen α

The slope safety factor is expressed as follow:

$$SF = \sum(i=1-n)\ Ti\ max / \sum(i=1-n) Ti\ mob$$

One can note that the safety factor SF, the variable to be determined, appears in the numerator too through the Tmax expression. Consequently one cannot directly determine SF.
Procedure to be adopted should be iterative, till obtaining convergence over a practically constant value of SF.
Steps are the following:

1.an initial value of SF is imposed (for example given by Fellenius method) and a first SF value is calculated;

2.the resulting new value of SF (SF') is compared to the initial value;

3.if the difference exceeds a prefixed value (e.g. SF'-SF> 0.001), one come back to step a), inserting, instead of the first value SF, the new calculated value;

4.if the difference lays inside the prexifed limit, calculation is aborted and SF' is the searched value.

Procedure generally needs from 4 to 8 iterations to converge.
The Bishop method requires the following two conditions be respected:

- s' = (g x h - gw x hw - c x tg α / SF)/(1+Y / SF) > 0

  with s' = normal stress on the slice bottom;

- $\cos \alpha$ x (1 + Y/SF) > 0.2.

If not, method may drive to unrealistic SF.
Method has preferably to be applied in case of homogeneous soil slope, both by lithological and geotechnical point of view.Use of this method in not recommended in case of highly overconsolidated layers.
Comparing simplified Bishop to its complete version, one can get a maximum difference in the SF values not exceeding 1%. With respect to other more rigorous methods, as G.L.E., deviation do not exceed 5%, except in case of SF<1, of scarce practical importance.

1.3.3 Simplified Janbu.

By the Bishop method the condition that the vertical forces working along the dividing surfaces of contiguous slices are negligible is imposed. Consequently slices interact among them through horizontal forces only.
This method, unlike the Bishop one, permits to verify potential sliding surfaces of poligonal shape. It is a method based on the equilibrium of the forces.
The maximum shearing resistance available along the potential sliding surface is given, for each slice, by

$$Ti\ max = Xi / (1+Yi/SF);$$

with Xi = [c+(g x h-gw x hw) x tg φ] x [1+(tg φ2)] x dx
where gw = water unit weight;
hw = height of the water table with respect to the slice bottom;
dx = horizontal length of the slice;
α = slice inclination with respect to horizontal plane.

Yi = tg α x tg φ

The mobilized shearing strenght along the shearing plane is, for each slice, given by:

$$Ti\ mob = Zi$$

with $Zi = g \times h \times dx \times tg\ \alpha$

The slope safety factor is expressed as follow:

$$SF = \sum(i=1-n) Ti\ max / \sum(i=1-n) Ti\ mob$$

One can note that the safety factor SF, the variable to be determined, appears in the numerator too through the Tmax expression. Consequently one cannot directly determine SF.
Procedure to be adopted should be iterative, till obtaining convergence over a practically constant value of SF.
Steps are the following:

1.an initial value of SF is imposed (for example given by Fellenius method) and a first SF value is calculated;

2.the resulting new value of SF (SF') is compared to the initial value;

3.if the difference exceeds a prefixed value (e.g. SF'-SF> 0.001), one come back to step a), inserting, instead of the first value SF, the new calculated value;

4.if the difference lays inside the prexifed limit, calculation is aborted and SF' is the searched value.

Procedure generally needs from 4 to 8 iterations to converge.
Method has preferably to be applied in case of heterogeneous soil slope, both by lithological and geotechnical point of view or in case of highly overconsolidated layers, where the potential sliding surface has probably an irregular shape, far from circularity.

The Janbu method might drive, with respect to more rigorous methods, as G.L.E., to not negligible difference in case of deep sliding surfaces or in presence of strong cohesion.

Then one suggests, in such a condition, inserting of a corrective factor that minimize the difference.
Janbu suggests the following form for this factor:

$$f = 1 + K \times [\ d/l\ - 1.4 \times (d/l)^2];$$

with
l = length of the straight line linking the two slope extremities;
d = maximum difference between the straight line linking the two slope extremities and the deepest point of the sliding surface, measured along the perpendicular direction;
K = constant equal to 0.31 in cohesionless soils (c=0) and to 0.5 in cohesive soils (c>0).

The corrected safety factor is therefore given by:

$$SF' = f \times SF$$

with SF = uncorrected safety factor.

1.3.4 Spencer

By the Spencer method the condition that the vertical forces working along the dividing surfaces of contiguous slices are parallel between them and applied to the meddle point on the bottom of the slice is imposed. It is, in its analytical expression, an extension of the Bishop method, and it is valid in case of sub-circular slip surfaces. It is a rigorous method for it is based on the equilibrium both of moments and forces.
The interaction force between contiguous slices applied in the middle point of the ith bottom slice is given by:

$$Qi = [(c \times l / SF) \times (W \cos \alpha - h \times gw \times l \times \sec \alpha) \times tg\, \varphi / SF - W\, sen\, \alpha] / (\cos (\alpha-\theta) \times ma$$

with $ma=1+ [tg\, \varphi \times tg(\alpha-\theta)] / SF$
θ = inclination of Qi force with respect to the horizontal plane.

Imposing equilibrium of the moments with respect to the centre of the arc drawn by the slip surface, one has:

$$(1) \sum Qi \times R \times \cos(\alpha-\theta)=0;$$

with R= radius of the circular arc.

Imposing equilibrium of the horizontal and vertical forces, one has respectively:

$$\sum Qi \cos \theta=0;$$

$$\sum Qi\, sen\, \theta=0.$$

Assuming that the forces Qi be parallel between them, one can also write:

$$(2) \sum Qi =0.$$

By this method two safety factors are calculated: the first one (Fsm) given by (1), for equilibrium of the moments, the second one (Fsf) by (2), for equilibrium of the forces. By a practical point of view, one procedes solving (1) and (2) for a given range of the angle teta, taking as safety factor the one for which be Fsm=FsF.

1.3.5 G.L.E. (General Limit Equilibrium)

This method (Fredlund e Kran, 1977) is a reformulation of the Morgenstern Price one. It is a rigorous method, for it takes in account both force and moment equilibrium.

Expression of the normal force working on the bottom of the i-th slice is the following:

$$N_i' = \frac{W_i + \left(X_{i-1} - X_i\right) + V_i - u_i l_i \cos\alpha_i - \frac{1}{F_s} c_i l_i \operatorname{sen}\alpha_i}{\cos\alpha_i + \frac{1}{F_s} \operatorname{sen}\alpha_i tg\varphi_i}$$

where:
W =weight of the slice;
X =vertical inter-slice forces;
V =external vertical forces;
u =hydraulic load;
l =length of the slice bottom;
α =inclination of the slice bottom.
Safety factor with respect to force equilibrium is given by:

$$F_{forze} = \frac{\sum(c_i l_i + N_i' tg\varphi_i)\cos\alpha_i}{\sum\left(N_i' + uil_i\right)\operatorname{sen}\alpha_i + \sum kW_i - \sum O_i}$$

where:
O = external horizontal forces;
k = horizontal seismic coefficient.
Safety factor with respect to moment equilibrium is given by:

$$F_{momenti} = \frac{\sum(c_i l_i + N_i' tg\varphi_i) r_i}{\sum W_i d_i - \sum\left(N_i' + uil_i\right)s_i + \sum kW_i m_i - \sum O_i n_i + \sum V_i d_i}$$

where:
r = distance, measured along perpendicular direction, from the slice bottom to the turning centre;
s = distance, measured along parallel direction, from the slice bottom to the turning centre;
d = distance, measured along parallel, from the slice bottom to the turning centre

m = distance, measured along vertical direction, from the slice centre of gravity to the turning centre;

n = distance, measured along vertical direction, from the topographic point lying on the vertical plane passing through the middle point of the bottom to the turning centre.

To these expressions one has to add that one linking the vertical (X) and horizontal (E) inter-slice forces:

$$X(x) = E(x)\lambda f(x)$$

where:

E = horizontal inter-slice forces;

λ = coefficient variable from 0 to 1;

f(x) = inter-slice function, imposed equal to 1 in the program.

Determination of the safety factor SF is given by the following calculation procedure.

- A first value of SF is given by, e.g, the Fellenius method.
- Varying the λ coefficient inside the interval 0-1 with a default step (e.g. 0.1), they are calculated, through a iterative procedure, the normal forces N', the inter-slice forces, starting from a null initial value (X=0, E=0) and finally the safety factors F_{forze} and $F_{momenti}$.
- Safey factor to adopt is that for which the calculated values of N', X and E give $F_{forze} = F_{momenti}$..

The choice of the f (x) function does not significantly affect the calculation. This method is valid both for circular and poligonal slip surfaces.

1.3.6 Sarma

The Sarma method differs from the above seen limit equilibrium methods for an approach not based on the asses of a safety factor, but rather on the calculation of the critical seismic coefficient for which the slope is in limit equilibrium (SF=1).

Inter-slice forces, in this case, are calculated through the expression:

$$X_{i-1} - X_i = \lambda \Psi_i;$$

where ψ_i is a force, selected by the user, such for which:

$$\sum \Psi_i = 0 \,.$$

The coefficient λ can be directly caculated by the formula::

$$\lambda = \frac{-D_i\left(y_i - y_g\right)}{\sum \Psi_i\left[\left(x_i - x_g\right) + \left(y_i - y_g\right)tg\left(\varphi_i - \alpha_i\right)\right]}$$

where:

D $\quad = W_i tg\left(\varphi_i - \alpha_i\right) + \dfrac{c_i b_i \cos\varphi_i \sec\alpha_i - u_i l_i \,\mathrm{sen}\,\varphi_i}{\cos\alpha_i \cos\varphi_i + \mathrm{sen}\,\alpha_i \,\mathrm{sen}\,\varphi_i}$;

b =length of the slice projected on the horizontal plane;

x_i, y_i = coordinates of the middle point of the slice bottom;

x_g, y_g = coordinates of the centre of gravity of the soil volume isolated by the slip surface.

Once the λ coefficient is determined, one can proceed to the direct calculation of the critical seismic coefficient:

$$k_c = \frac{\sum D_i + \lambda \sum \psi_i tg\left(\varphi_i - \alpha_i\right)}{\sum W_i}$$

The determined Kc value consequently represents the seismic coefficient to which a safety factor SF=1 is associated.

More problematic is the reverse calculation, that is, be Kc known, that might be also imposed equal to zero, determine the associated safety factor.

This is the suggested procedure.

- Kc in critical condition (SF=1) is determined.
- An initial value of SF is imposed, e.g. 1.3, and the calculation of Kc is repeated, using values of the angle of shearing resistance and cohesion corrected as follow:

$$c_c = \frac{c}{F_s}, \varphi_c = \frac{\varphi}{F_s} \,.$$

- Repeating the procedure with new values of SF and drawing a chart with SF along the X axis and Kc along the Y axis, one can immediately get the safety factor associated to each Kc value.

Actually this chart is not linear, then interpolation might drive to an error often not negligible. One suggests to use this method exclusively to determine Kc in critical condition when operating in seismic zone.

1.3.7 Assessing force deficit.

Taking in account the shifting of the soil volume bordered by the slipping surface, one can assess the force to be applied to reach equilibrium, that is the SF=1 condition (force deficit).
At the equilibrium of the horizontal shifting, one has for the i-th slice:

$$\Delta E_i = (c_i \Delta l_i + N_i tg\varphi_i)\cos\alpha - (N_i + U_i) sen\alpha$$

If the soil volume is at the equilibrium (SF=1), the sum of the horizontal forces extended to all the slices must to be null.

$$\sum \Delta E_i = 0$$

In the hypothesis SF<1 the sum gives the value of the horizontal force to be applied to reach equilibrium.

1.4 Selecting the geotechnical parameters to be used in the slope analysis.

1.4.1 Granular soils.

In sandy or gravelly soils one has to be used either the peak or critical angle of shearing strength, as a function of the relative density of the layer.
In case of relative density lesser than 20%, one suggests to employ the critical angle of shearing strength, given, as a first approximation, from the peak value through the formula by Terzaghi:

$$tg\ \varphi' = 2/3 \times tg\ \varphi;$$

with φ'= critical angle of shearing strength;
φ = peak angle of shearing strength.

In case of relative density higher than 70% the peak angle of of shearing strength has to be used. In the intermediate cases one has to be interpolate between the two limit values (phi' e phi).
For slope stability analysis along slip surfaces still moving one has to be used the constant volume angle of shearing strength.
In case of analysis in undrained condition (e.g. for temporary excavations), one should take in account of the weak cohesion due to capillary tension.

1.4.2 Cohesive soils.

The slope stability analysis have always to be executed, except some particularly cases, in long term conditions, which are the more unfavourable. One has consequently to be used, as geotechnical parameters, angle of shearing strength and drained cohesion.
In case of slope composed by normally consolidated cohesive layers, inside new-formed landslides, one has to be used the angle of shearing strength only (the drained cohesion be in this case null).
In presence of overconsolidated, not fractured, cohesive layers, one has to be used instead both the angle of shearing strength and the drained cohesion.

In case of diffuse fractures, a long term reduction of the cohesion has to be hypotized, that has consequently to be neglected.
For slope stability analysis in presence of past landslides, one has to be used the angle of shearing strength only, imposing a null cohesion.
In case of analysis in undrained condition (e.g. for temporary excavations), one should take in account of the undrained cohesion only, imposing a null angle of shearing strength.

1.5 Effects of external loads and slope stabilization techniques.

1.5.1 External loads.

With Sn we mark the normal component, with respect to the potential slide plane, of the sum of the applied forces on the slice bottom by external loads (Si). Its expression is the following:

$$Sn = Si\ (\text{sen}\,\beta \cos\alpha + \cos\beta \sin\alpha);$$

where
α=inclination of the bottom slice.
β=inclination of the load with respect to the horizontal plane, increasing counterclockwise.

With St we instead mark the tangential component, with respect to the potential slide plane, of the sum of the applied forces on the slice bottom by external loads (Si). Its expression is the following:

$$St = Si\ (\cos\beta\ \cos\alpha - \text{sen}\,\beta\ \text{sen}\,\alpha);$$

There is consequently a dual effect of external load upon the soil slope: there is a positive or negative variation (depending on the inclination of the load with respect to the potential slide plane) of both normal and tangential forces, thereby changing the values of the maximum stress and strength.

$$F_s = \frac{Forze_{stab} + \sum Sn_i}{Forze_{instab} + \sum St_i}$$

1.5.2 Seismic forces.

Analysis of the effect of seismic forces upon the global stability of a soil slope can be executed through two different approaches.

1. by introducing a simplification based on the concept that the seism acts as a force system with constant module and direction all earthquake long (pseudostatic method);

2. by introducing a force system which takes in account of their variation in module and direction during the seismic event (dynamic method).

The second procedure (dynamic method) needs of a recorded or simulated accelerogram, which gives, for each instant, trend of accelerations undergo by the soil slope (see paragraph 1.10).

The pseudostatic method requests, as input datum, the peak horizontal seismic acceleration only. The a_g value (peak horizontal seismic acceleration) is given by the following expression:

$$a_g = S_s S_t a_{bedrock}$$

where $a_{bedrock}$ is the peak horizontal acceleration at the bedrock and S_s and S_t respectively the stratigraphic and topographic factors of seismic amplification.
An assessment of agv (peak vertical seismic acceleration) can be given as suggested by Tezcan et alii (1971):

$$agv = f \text{ x } ag;$$

with f = factor generally ranging from 0.5 to 0.67.
An evaluation of the seismic effect with regard to the slope stability can be executed, supposing that, during the seismc event, an horizontal force, acting in the centre of gravity of every slice and direct outward, be applied with a module given by:

$$F_{sisma} = k_c W_i$$

with kc = horizontal seismic coefficient = β ag, β= reduction factor;
W = weight of the i-th slice.

Taking in account the vertical seismic force too, the slice weight has to be modified according to the following expression:

$$W_{is} = W_i \left(1 \pm k_v\right)$$

where kv is the vertical seismic coefficient, usually imposed equal to 0.5 kc. Estimating the safety factor, the calculated seismic force has to be added to the unstable forces.

$$F_s = \frac{Forze_{stab}}{Forze_{instab} + \sum F_{sisma} \cos \alpha_i}$$

1.5.3 Tiebacks.

Tiebacking of a potential unstable slope try to get a dual target: adding tangential force (St) in opposition to the unstable forces, caused by gravity and seism, and increasing the normal forces (Sn) acting on the slice bottom. We can distingue among passive, active and partially active tiebacks.

PASSIVE TIEBACK

In this case the anchorage is not pre-stressed. The effect is a long-term increasing of the stabling normal forces acting on the slide plane, caused by deformation of the upslope soil volume.

By an analytical poitn of view, the safety factor can be expressed as follow:

$$F_s = \frac{Forze_{stab} + \sum T \cos \theta_i}{Forze_{instab}}$$

T= pull-out resistance of the tieback;

θ= angle between the tieback and the normal to the slice bottom, where it is applied.

ACTIVE TIEBACK

In this case the tieback is pre-stressed upto reaching the pull-out resistance. The effect is an immediately contrasting of the tangetial forces acting along the slide plane.

By an analytical poitn of view, the safety factor can be expressed as follow:

$$F_s = \frac{Forze_{stab}}{Forze_{instab} - \sum Tsin\theta_i}$$

T= prestressing force of the tieback;

θ= angle between the tieback and the normal to the slice bottom, where it is applied.

PARTIALLY ACTIVE TIEBACK

In this case the tieback is pre-stressed upto a value lesser than the pull-out resistance. The effects are an immediately contrasting of the tangetial forces acting along the slide plane, with a force equal to prestressing one, and a long-term increasing of the stabling normal forces acting on the slide plane, caused by deformation of the upslope soil volume, with force given by the difference between the pull-out force and the prestressing one.

By an analytical poitn of view, the safety factor can be expressed as follow:

$$F_s = \frac{Forze_{stab} + \sum(T-P)\cos\theta_i}{Forze_{instab} - \sum Psin\theta_i}$$

T=pull-out resistance of the tieback;

P=prestressing force of the tieback;

θ=angle between the tieback and the normal to the slice bottom, where it is applied.

The T value can be calculated through the formulas by Schneebeli and Bustamante Doix.

Schneebeli

In case of granular soil (φ>0) the formula is the following:

$$T_l = \pi D_p Ltg\left(45 - \frac{\varphi}{2}\right)\operatorname{sen}\varphi \frac{1+e^{2\pi tg\varphi}}{2}\gamma Z \;;$$

where:
D_p =borehole diameter;
L =length of the bond;
Z =depth of the median point of the bond;
γ =unit weight of the soil above the bond.
In case of cohesive soil in undrained condition (φ=0) one can instead use the expression:

$$T_l = \pi D_p L c$$

c= undrained cohesion of the soil along the anchored bulge.
Pull-out resistance is given by the ratio between Tl and a safety factor, usually chosen equal to 2.5:

$$T = \frac{T_l}{2,5}$$

Bustamante Doix

The expression is the following:

$$T_l = \pi \alpha D_p L q_s ;$$

dove:
D_p =borehole diameter;
L =length of the bond;
α =factor which measures increasing of the bond diameter.
qs =lateral friction or adhesion along the bond.

The α coefficient is function of the prevalent lithology along the bond and of the grouting method. One can assess it by the following table:

Lithology	Coefficient α	
	repeated grouting	*simple grouting*
Gravel	1.8	1.3-1.4
Sandy gravel	1.6-1.8	1.2-1.4
Gravely sand	1.5-1.6	1.2-1.3
Clean sand	1.4-1.5	1.1-1.2
Silty sand	1.4-1.5	1.1-1.2

Silt	1.4-1.6	1.1-1.2
Clay	1.8-2.0	1.2
Marl and sandstone weathered and/or fractured	1.8	1.1-1.2

The qs factor can be obtained by these formulas:

simple grouting :

$q_s(MPa) = 0{,}01(Dr - 50) + 0{,}05$ granular soil (Dr=relative density)

$q_s(MPa) = 0{,}006(c - 10) + 0{,}1$ cohesive soil (c=cohesion in t/mq)

repeated grouting :

$q_s(MPa) = 0{,}01(Dr - 50) + 0{,}1$ granular soil(Dr=relative density)

$q_s(MPa) = 0{,}008(c - 10) + 0{,}18$ cohesive soil(c=cohesion in t/mq)

Pull-out resistance is given then by the ratio between T1 and a safety factor, usually posed equal to 2.5

$$T = \frac{T_l}{2{,}5}$$

Placing and sizing tiebacks have to be executed taking in account that:

- bond has to be placed at a depth higher than the potential sliding plane to perform its stabilizing action;
- optimum inclination of the tieback can be assess by the formula:

$$i_{optimum} = \mathrm{arctg}(\tan \varphi/Fs)$$

where:
φ=angle of shear resistance of the soil;
Fs=safety factor.

1.5.4 Lattice of micropiles

Stabilization of a soil slope can be achieved through lattices of small-diameter piles (micropiles). The effect that one trys to get in this case is to increase the shear resistance along the slide plane by a pile-soil complex which behaves as an homogenous system with respect to the forces acting on the slope. This action of slope reinforcement can be taken in account in the stability calculation, supposing a virtual increasing of mechanical resistance of the soil composing the slope.
Supposing, acting on the safety side, that the angle of shear resistance of the soil rests invaried, one can take in account hte improvment of the mechanical resistance of the slope, considering increasing cohesion.
The procedure is depicted as follow:

- The equivalent strength area of the micropile is calculated through the relation:

  $$Ae = Acon + Co \times Asteel;$$

  with
  Acon=transversal sectional area of the micropile;
  Asteel=area of tubular steel casing;
  Co=homogenization coefficient.
- Increasing of the potential slide plane is calculated as follow:

  $$DS=Co \times Nm \times Ae;$$

  with
  Co=ratio between the elastic modulus of pile and of soil:

  $$Co=Ep/Et;$$

  where:
  Ep=elastic modulus of concrete;
  Et=elastic modulus of soil;

  Nm=number of micropile rows per meter along the vertical direction.

- Increasing of cohesion is determined along the potential slide plane by the expression:

$$Dc = (ci + Smi \times \tan phii) \times DS / \sum li$$

where:
ci=average cohesion along the i-th slice;
phii=average angle of shear resistance along the i-th slice;
Smi=average lithostatic pressure acting along the i-th slice;
∑li=sum of the lengths of all the slice bottoms.

- Virtual cohesion is finally assessed for each slice by the relation:

$$Cv = Ci + Dc.$$

To have a stabilizing effect the lattice of micropiles has to be founded to a depth higher than the potential slide plane.

1.5.5 Retaining and gabion walls.

Shallow stabilizing works, as retaining and gabion walls, has to be considered, in the slope stability, both for their effect as vertical load and for stabilizing action on the upslope soil volume.
The two effects has to be calculated as follow:

- vertical load (Sv) is given by the weight of the wall per meter of length;
- maximum retaining force (So), supposing acting on the horizontal plane, is given by:

$$S_o = (W_{muro} + S_a) tg\varphi + cb \text{ ;}$$

where:
W_{muro} = wall weight per meter of lenght;

b = width of the wall bottom;

S_a = vertical active soil pressure;

The maximum retaining force has to be only taken in account when the potential slide plane intersects the wall: in case of deeper sliding surfaces the wall acting as a vertical load only, without performs retaining effect.
The effects, in the slope stability calculation, is of changing the stabilizing and unstabilizing forces acting along the slice bottoms.
By an analytical point of view this can be expressed as follow:

- in case of wall acting as vertical load only:

$$F_s = \frac{Forze_{stab} + \sum Sv\cos\alpha_i}{Forze_{instab} + \sum Sv sin\alpha_i}$$

- in case of wall acting as retaining force:

$$F_s = \frac{Forze_{stab} + \sum So sin\alpha_i}{Forze_{instab} + \sum So\cos\alpha_i}$$

1.5.6 Pile groups

Group of big-size piles, which can resist to horizontal forces, can be used for soil slope stabilization.The retaining action can be calculated considering the effect of a single pile first and then of the group of piles.

1.5.6.1 Single pile

We consider the Broms theory (1964), applied to stiff pile with locked head, distinguishing between piles founded in cohesive and pile founded in granular soils.

1.5.6.1.1 Cohesive soil

Lateral resistance is given by:

$$Rlat=9\ Cu\ Dpile\ (Lpile - 1.5\ Dpile);$$

with
Cu=soil undrained cohesion;
Dpile=mean diameter or size of the pile;
Lpile=length of the pile.

Soil response has consequently a rectangular shape, then uniform in depth.

$$Rz=9\ Cu\ Dpile.$$

1.5.6.1.2 Granular soil.

In this case the expression in the following:

$$Rlat=1.5\ \gamma Lpile^2\ Dpile\ Kp;$$

with
γ = soil unit weight;
Kp=(1 + sen φ)/(1 - sen φ).

Soil response has consequently a triangular shape, then linearly increasing in depth.

$$Rz=3\ \gamma\ Lpile\ Dpile\ Kp.$$

1.5.6.2 Group effect

Efficiency of a group of piles undergone to horizontal loads is defined as the ratio between horizontal bearing capacity of the group and the sum of the horizontal bearing capacity of each piles.
In granular soils efficiency is often close to 1, in cohesive soils is generally lesser than 1.

It generally suggests to take in account, as horizontal bearing capacity of the group, of the lesser between these two values.

1. sum of the horizontal bearing capacity of each piles.
2. the horizontal bearing capacity of a foundation block with width equal to the width of the pile group (side of the pile group perpendicolar to the load direction) and with a thickness equal to the pile lenghts, that is:
3.

1.5.6.2.1 Cohesive soil:

$$Rgroup=9\ Cu\ Lpile(Lgroup-Cr);$$

with
Lpile=width of the pile group;
Cr=the lesser between (1.5Dpile) and (0.1Lpile).

1.5.6.2.2 Granular soil:

$$Rgroup=1.5\ \gamma\ Lpile^2 Lgroup\ Kp.$$

1.5.6.3 Stabilizing effect of a pile group

The stabilizing effect of a pile group in a soil slope consists of reducing the unstabilizing forces acting on the slice bottom.

$$F_s = \frac{Forze_{stab}}{Forze_{instab} - \sum R_{palificata} \cos\alpha_i}$$

with
Rpalificata = horizontal bearing capacity of the pile group;
α = inclination of the slice bottom.

1.5.7 Reinforced earth wall

A reinforced earth wall, as to the assessment of the retaining effect on the soil upslope volume, exactly behaves as a concrete wall or a gabion wall.
In addition, however, one needs to verify the internal stability, then to check that the stress acting on the single reinforcement does not exceed its mechanical strength. The single reinforcement, intersecting the potential slide plane, isolates a soil wedge upslope, which tends to move outward. The reinforcement contrasts sliding, generating, along the soil-reinforcement contact surface , a friction force inward.
By an analytical point of view, this force can be expressed as follow:

$$Fr = Cf\ \ tg\ \varphi\ \ Lg\ \ sv\ \ Lf\ /\ Fsg$$

with
Cf = soil-reinforcement friction coefficient (usually ranging from 0.5 to 1.0);
φ = angle of shearing resistance of the soil;
Lg = reinforcement lenght (in this case imposed equal to 1);
sv = lithostatic effective pressure acting on the reinforcement;
Lf = reinforcement tract between the sliding plane and the ground (tract along which the friction force is generated);
Fsg = safety factor (usually equal to 1.5).

Fr cannot be higher than the tensile strength of the single reinforcement, otherwise it could pull out and the wall will collapse.

1.5.8 Tension crack

In case of incipient or in evolution landslides, they frequently forms on the ground tension cracks. These, beside to represent preferential paths of infiltration of water by rainfall or runoff in the landslide volume, could conduct to the settling of ponds, acting as load on the slope.
Normal and tangential forces on the slice bottom are given by:

$$F_s = \frac{Forze_{stab} + \sum Sv \cos\alpha_i}{Forze_{instab} + \sum Sv sin\alpha_i}$$

with
Sv= γ h
γ = unit weight of water;
h = water depth inside the tension crack;

Shallow tension cracks can also form in cohesive soils by drying.

1.5.9 Diaphragm wall.

Diaphragm wall is a is a continuous wall of elevate thickness which contrasts the landslide through the passive soil strength.
Its stabilizing effect can be analytically express as follow:

$$F_s = \frac{Forze_{stab} + \sum S_{passiva} sin(\alpha_i + \delta)}{Forze_{instab}}$$

where
Spassiva = passive soil pressure downslope;
α = inclination of the slice bottom;
δ= inclination of the resultant of the passive pressure; this parameter can assume as maximum value δ=arctg[(2/3)tg(φ)], where φ is the angle of shearing resistance of the soil.

In case the slide plane passes below the diaphragm wall, this will behave as a shallow load.

$$F_s = \frac{Forze_{stab} + \sum Sv \cos\alpha_i}{Forze_{instab} + \sum Sv sin\alpha_i}$$

with
Sv=diaphragm wall weight.

1.5.10 Soil nails.

The presence of a structural linear steel element (tieback or nail) acts as resistive section to the pure shear along the potential slide plane. In this case, to the aim of representing the stress resistance to the pure shear by the element, from now on called “nail”, one can insert an increasing of resistance (T_d) inside the soil due to the so-called Dowel effect.
Conditions for the Dowel effect are:

- elements have to be passive, that is not prestressed;
- adequate stiffness and stength of the soil around the grout-nail complex; sufficiente rigidezza e resistenza del terreno al contorno dell’insieme cementazione-chiodo.

As suggested by Bjurstrom, this effect depends on three parameters:

1. nail or bar diameter (**$\varnothing_b$** o **d_b**)
2. the minimum value between the uniaxial compressive strength of the grout and the soil one (**σ_c**)
3. the yielding strength of the nail or bar (**σ_s**)

The contribution to the resistance force by each nail is equal to:

$$T_d = 0{,}67 d_b^{\,2} \sqrt{\sigma_s \sigma_c} \text{ [MPa * m}^2\text{] ;}$$

where the parameter are expressed in:
d_b [m], σ_c [MPa], σ_s [MPa]
The cohesion increment due to the Dowel effect C_{dis} given by:

$$C_d = \frac{nT_d}{s} \qquad [\mathbf{MPa}];$$

where :
n = number of nails crossing the slide plane
s = length of the considered slide plane.

To insert increasing of resistance due to this effect in a slope stbility calculation, it needs to calculate parameter $\boldsymbol{n_i}$ e $\boldsymbol{s_i}$ for each slice bottom of the potential slide plane and assesses the cohesion due to the Dowel effect to be summed to the soil one.

1.6 Effect of water over slope stability

Water table works in the calculation in two different ways:

- through the introduction of a hydrostatic load, diminishing normal forces acting on the slice bottom.
- through using in slope analysis the saturated unit weight of the soil.

In case of shallow hydraulic loads (water course, pond etc) to improve safety ground of the slope can be considered permeable. This involves the slope has to be considered saturated and consequently behaving as in presence of a water table. This, by the calculation point of view, takes to a partial compensation of the effect, generally stabilizing, if applied to the bottom of the slope, of the shallow hydraulic loads.
A particular case is that one where the water circulation is bounded inside a layer of high permeability with limited thickness and where a real water table consequently cannot be individuated. In this condition imposing a continuous water table might conduct to gross errors. These errors, in case of layer with phi>0 (drained condition) are usually on the side of safety, but in layers with phi=0 (undrained condition) on the contrary are on the side of unsafety.
In principle, one should consider the force exerted by water in its seepage motion inside the slope too. This force can be introduced in the calculation as equivalent depth fo water to be summed to the static water depth. Contribution can be assess by the expression:

$$h(m) = \frac{v^2}{2g}$$

where:

v(m/s)=	Seepage velocity of water inside the slope, given by the Darcy law: k x i;
k(m/s)=	Soil permeability;
i=	Hydraulic gradient
g(m/s2)=	Gravity acceleration

It is evident that, being seepage velocity very low, often this effect can be neglected.

An alternative way to introduce in the calculation the effect of the water presence inside the slope is to define a coefficient of neutral pressure r_u. This parameter can be defined as follow:

$$r_u = \frac{\gamma_w h_w}{\gamma h}$$

where:

γ_w(t/mc)=	Water unit weight;
γ(t/mc)=	Soil unit weight;
h_w(m)=	Depth of water table with respect to the slice bottom;
h(m)=	Thickness of the slice.

The hydraulic load acting on the bottom of the i-th slice is given by:

$$u_i = r_u y_i h_i$$

In presence of seismic force has also to be evaluated increasing of neutral pressure Δu products by strains caused by the seismic waves in saturated layers. Inside granular soils one can applied relationship by Seed e Booker (1977), where the Δu variation is function of number of load cycles N caused by the seism.

$$\Delta u_N = \sigma'_0 \frac{2}{\pi} sen^{-1}\left[\left(\frac{N}{N_L}\right)^{0.5a}\right]$$

where N_L is the number of load cycles necessary to produce liquefaction in the saturated soil layer, σ'_0 is the effective mean pressure in static condition and *a* is a factor associated to the relative density, expressed in decimal format, through the relation (Fardis e Veneziano, 1981):

$$a=0.96D_r^{0.83}$$

the parameter N_L can be, approximately, given by the following table (Seed et al., 1975):

N_L	τ_{medio}/σ'_0
100	0.09
30	0.12
10	0.15
3	0.22

where τ_{medio} is the mean shear stress due to the seism, usually sets equal to 65% of τ_{max}.
N can be correlated to the magnitude through the following table (Seed et al., 1975):

N	M
3.8	5
4.0	5.5
4.5	6.0
7.0	6.5
10.0	7

A simply relation, which permits to estimate approximately the increment of the pore pressure Δu in seismic condition inside a cohesive soil, is given by Matsui et al. (1980):

$$\Delta u = \sigma'_0 \, \beta Log_{10}\left(\frac{\gamma_{max}}{\gamma_s}\right)$$

where:

σ'_0 = effective mean pressure given by:

$$\sigma'_0 = \sigma'_{v0}\,\frac{1+2k_0}{3}$$

where σ'_{v0} is the effective vertical pressure and k_0 is the at rest stress ratio, calculable through the formula:

$$k_0 = 1 - sen\varphi$$

being φ the angle of shear resistance of the soil layer;

β = empirical factor equal to 0.45;

γ_s = strain associated to the volumetric threshold, estimable by the formula:

$$\gamma_s = A(OCR - 1) + B)$$

where A and B are given as a function of the plasticity index IP:
IP= 20% - A=0.4 10^{-3} B=0.6 10^{-3}
IP= 40% - A=1.2 10^{-3} B=1.1 10^{-3}
IP= 55% - A=2.5 10^{-3} B=1.2 10^{-3}

γ_{max} = maximum soil strain due to the seism, given by:

$$\gamma_{\max} = \frac{\tau_{\max}}{G_\gamma}$$

where τ_{max} is the maximum shear stress produced by the eathquake:

$$\tau_{\max} = a_g \sigma_v r_d$$

with a_g the seismic acceleration (g), σ_v the total vertical pressure and r_d = 1-0.015Z, where Z is the depth to the ground surface; G_γ is the shear modulus associated to the strain γ; G_γ , for high strain, is always lesser than G_0, modulus associated to low strain, and, in case of γ values close to the volumetric threshold, it can be setted approximately close to 0.75 G_0; it reminds G_0 is correlable to the S wave velocity by the analytical expression:

$$G_0 = \rho V_s^{\,2}$$

where ρ is the mass density of the soil layer given by the ratio between the soil unit weight and the acceleration of gravity (9,81 m/s^2).

The Matsui relation involves that, to have a positive raising of the pore pressure, γ_{max} must be higher than γ_s. This means that important increments of Δu can be reached only in case of high seismic forces in soil layers having low G0 values.
In a cohesive soil with plasticity index less than or equal to 55%(IP≤55%), in undrained condition it can be observed a decreasing of cu (undrained cohesion) due to the load cycles caused by the seism. The degradation index can be estimated, in case where $\Delta u / \sigma'_{v0} > 0.5$, by the following relation:

$$\delta c_u = \sqrt{\frac{c_u - \Delta u}{c_u}}$$

where instead it's $\Delta u / \sigma'_{v0} \leq 0.5$ δc_u can be calculated by:

$$\delta c_u = N^{-t}$$

where N is the number of load cycles produced by the earthquake (see above) and t is a parameter function of the cyclic shear strain associated to the seism $\gamma_c = \frac{\tau_{media}}{G_\gamma}$ and of the strain linked to the volumetric threshold(see above):

$$t = s(\gamma_c - \gamma_s)^r$$

The parameters s and r are given by the following table:(Matasovic, 1993):

	OCR=1			OCR=2	OCR=4
	IP=15%	IP=30%	IP=50%	IP=50%	IP=50%
s	0.195	0.095	0.075	0.054	0.042
r	0.600	0.600	0.495	0.480	0.423

Finally the corrected undrained cohesion is calculable by the following formula:

$$c_{uc} = \delta c_u x c_u$$

1.7 Probabilistic analysis method.

1.7.1 Introduction.

In a slope stability analysis the equilibrium condition is reached, when the ratio between stabilizing (R)and unstabilizing (S) forces is equal to 1 (R/S=1). The ratio R/S, as it knows, is called safety factor (SF). To take in account possible errors introduced in the analysis, however, one takes as reference a value of the safety factor higher than 1, usually 1.3.
Sources of errors are generally four:

1. the natural inhomogeneity of the soil layers: geognostic survey are usually punctual and they do not allow, or partially allow, to individuate lateral variability of the mechanical characteristics of the subsoil;
2. the inaccuracy in the execution of geotechnical survey performed in situ or in laboratory;
3. the approximation of the empirical correlations available to indirectly obtain the parameters of the soil;
4. simplification introduced in the subsoil model.

In a deterministic approach errors introduced in the calculation are taken in account imposing a safety factor higher than 1.
A probabilistic analysis, which allows to manage errors through theory of probability, permits to face the problem of the sources of uncertainty in a more rigorous and rational way.
The probabilistic analysis deserts the concept of safety factor, replacing it with the **safety margin** (MS) one, defined as the difference between the stabilizing and the destabilizing forces (MS=R-S). As the rigorous application, however, of this definition does not allow using of some calculation methods, as the Sarma and Spencer ones, the safety margin is often defined as follow:

$$MS = \frac{R}{S} - 1 = Fs - 1$$

By a practical point of view this definition of the safety margin is obtained dividing the two members by S, giving a normalized MS.
At the equilibrium MS has to be equal to zero (S=R), values higher than zero indicates stable slope, values lower than zero indicates unstable slope.

The sources of uncertainty take to generation of a range of MS values distributed after a probability density function (e.g. the gaussian one).
The probability of collapse (pr) is defined by the probability that the MS value be lower than 0 (equilibrium condition). The reliability index is linked to the probability of collapse by the formula:

$$I = 1 - pr$$

The slope stability analysis by a probabilistic approach are usually executed by the Montecarlo methods. Such a method allows processing a MS distribution, starting from a relatively limited number of couple of measured c e φ values.
Upon this distribution one will be able to execute an assess of the probability of collapse.

1.7.2 Montecarlo methods applied to a slope stability problem.

Montecarlo methods are based on the generation of random numbers, chosen inside specific range, which have statistical properties. Among the several possible application of these methods, there is that one called 'of sampling' which consists in deducting general propierties of a big set, examining a random subset of it only, considerated representative of the set itself. Obviously bigger the size of the random sample, more representative the deducted properties.
In case of application of the methods to a slope stability analysis, the procedure to be adopt could be the following:

- a distribution of the aleatory variables cohesion and angle of shearing resistance, measured in situ or in laboratory, is generated, supposing that it is of gaussian kind;
- through a random number generator, a set of value ranging between 0 and 1 are created;
- one associates to each random numeric value of the set a value of cohesion and angle of shearing resistance, based on the probability distribution curve of these two variables (then by making sure that the frequency with which a specific parameter is used in the calculation be equal to its probability obtained by the gaussian curve

of probability of the parameter itself); in this way the set of random numbers, previously generated, is turn in a set of couples of c and φ values;

- chosen a method of calculation, the verification is executed for each couple of c and φ values generated, obtaining the respective safety margin MS;
- the distribution curve of frequency of the calculated MS values is processed, e.g. in the form of histogram, showing the trend of these coefficients.

After processed a stable probability curve, therefore one can calculate the average value of MS (Msm) and the medium square deviation sMS of the processed virtual sample.
The Montecarlo methods can be used for slope stability back analysis.
Infact, making a set of hypothetical curves of distribution of c and φ, one can assess for which range of these values the slope is stable. Confrontation between the hypothetical distribution of the geotechnical parameters and the measured one allows to draw conclusion about the slope stability.
Montecarlo methods needs, to allow obtaining valid distribution of MS, an enough high number of c and φ couples. To get stable distribution of MS are usually needed some thousands verifications. Reaching the stability of the distribution curves can be evaluated, applying the Montecarlo methods on two different set of verifications and comparing the respective distributions by the $\chi 2$ test.

1.7.3 Assessment of the collapse probabilty.

Task of an analysis executed by the probabilistic criterion is to calculate the collapse probability (pr) of the examined slope. Montecarlo methods allow to assess reliable evaluation of the average value Msm and of the medium square deviation of the safety margin. These variable allow to directly obtain the MS value associated to a specific exceedance probability (characteristic value of MS) through the relationship:

$$MS_k = MS_m(1 + \chi V_{ms})$$

where:

MS_k = characteristic value of MS;
MS_m = mean value of MS;
V_{ms} = coefficient of variation of MS, defined as the ratio between the medium square deviation and the mean value of MS;
χ = parameter depending on the adopted function of probability distribution and of probability of exceedance.

Probability of exceedance is defined as the probability that the 'actual' value of MS be lesser than a given value.

Parameter χ depend exclusively on the probability density function chosen. In case of gaussian distribution the χ values can be directly obtained by the following table:

Table 1

Probability of exceedance %	χ
1	-2.326
5	-1.645
10	-1.282
20	-0.842
30	-0.524
40	-0.253
50	0
60	0.253
70	0.524
80	0.842
90	1.282
95	1.645
99	2.326

From the definition of exceedance probability directly derives the collapse one, which can be defined as the probability of exceedance associated to a null value of MS. The parameter χ associated to a value of MS=0 is given by the formula:

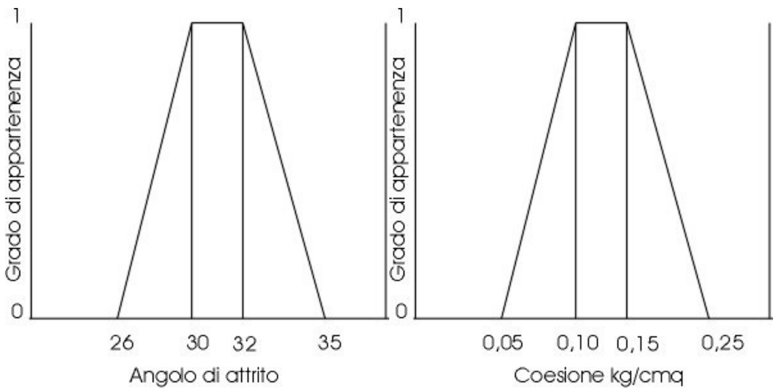
$$\chi_r = -\frac{1}{V_{ms}}$$

Once calculated the χ_r value, from table 1 the collapse probability is given.
One has to be assess which collpase probability may be considered acceptable, that is for which value of pr the slope can be defined stable. Such a value should be in principle associated to the importance of the site and to the state of knowledge of the soil characteristic. Taking as reference what suggests by Priest and Brown (1998), one can approximately consider as reference probability a 1% value in the case in which the landslide would not cause significant damages and a 0.3% otherwise.
Consequently, if the collapse probability were lesser than this value, the slope can be considered stable and unstable otherwise.

1.8 Qualitative analysis of slope stability by the fuzzy methodAnalisi qualitativa del grado di stabilità di un pendio con il metodo fuzzy.

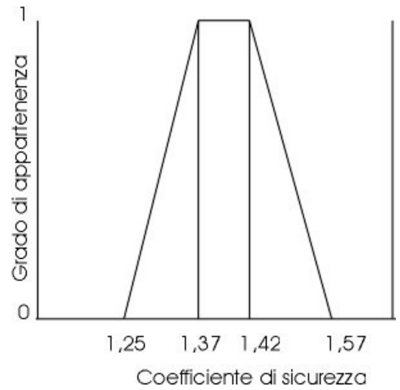
The fuzzy logic allows to face in a rigorous way problems in which there is the need to get a qualitative judgment, starting from a set of data known with a high uncertainty only. Among the numerous applications, there is that one relative to the evaluation of the stability level of a slope.
The calculation procedure consist of 3 fundamental steps:

1. Definition of the membership functions of the parameters cohesion and angle of shearing resistance. The membership function is the basic tool of the fuzzy logic. It generally point out the degree of membership of a subset of data to a specific set. In this specific case, it represents the degree of membership of the parameters of shear strength c, φ to the layer taken in account. Those values c, φ, which are surely members of the data set of the considered layer, are marked by a degree of membership equal to 1. Those values c, φ, which are surely not members of the data set of the considered layer, are marked by a degree of membership equal to 0. Intermediate values have intermediate degree of membership.

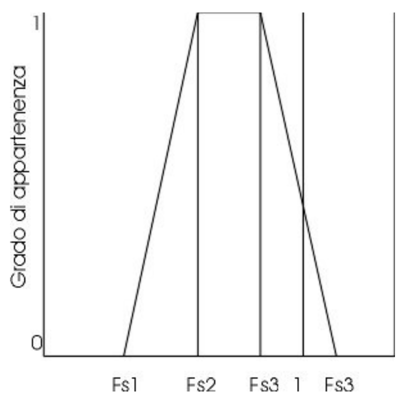


2. Making of the membership function of the safety factor. Merging together the couples of value c, φ and using a deterministic method of calculation (Fellenius, Bishop, Janbu, etc.) the correspondent value of

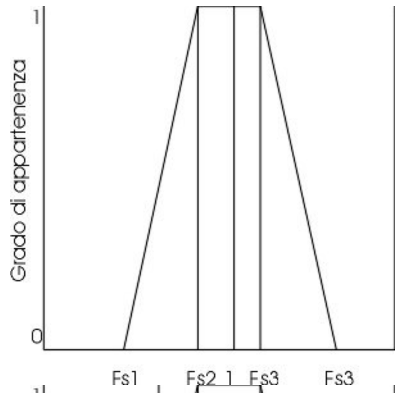
SF is calculated, one for each couple. Through the SF values so calculated, the membership function of the safety factors is drawn.



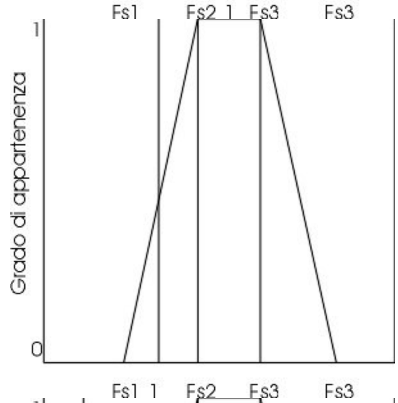
3. Qualitative judgement about the slope stability. Taken as reference a factor of safety equal to 1 (equilibrium condition), one can get a qualitative assessment of the degree of stability of the examined slope. Sakurai e Shimizu (1987) suggest the following scheme:



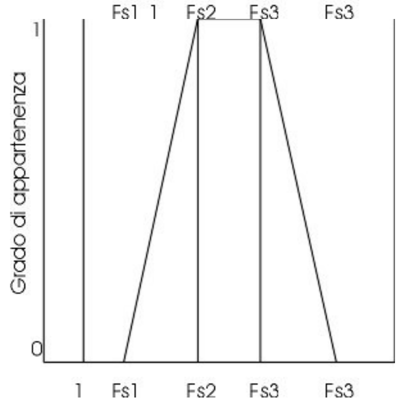
Unstable slope



Fairly stable slope



Medium stable slope



Stable slope

1.9 Slope stability analysis by the infinite slope method.

Method of infinite slope is an expeditious procedure of calculation to verify the stability of slopes with homogeneous geometrical and stratigraphic characteristics. It is particularly fitted to assess the stability of a soil layer overlying a bedrock or a more consolidated layer.
The generale calculation relationship is the following:

$$F_s = \frac{\dfrac{cb}{\cos\alpha} + \left[(W + Qb)\cos\alpha - U\right]\tan\varphi}{(W + Qb)sen\alpha + \dfrac{a_g}{2}W\cos\alpha}$$

where:

c = cohesion of the sliding layer;

b = total length of the verification section;

α = inclination of the slope;

W = weight of the sliding layer = γz b (γ=soil unit weight, z=layer thickness);

U = hydrostatic pressure = $\gamma_w z_w b\cos\alpha$ (γ_w =water unit weight, z_w=thickness of the water column); if z_w>z U has to be imposed equal to 0.

φ = angle of shearing resistance of the sliding layer;

a_g = horizontal seismic acceleration;

Q = shallow load.

1.10 Analysis of slope stability by simplified dynamic method (Newmark and Newmark modified)

Simplified dynamic analysis of a slope allows to calculate the cumulative displacements produced by an earthquake, through the processing of its accelerogram, inside a potentially unstable slope. The displacement method, originally developed by Newmark (1965) involves they are satisfied the following conditions:

- the reference accelerogram be valid for the whole examined slope;
- the shear resistance of the soil layers be the same both in static and dynamic conditions;
- the potential landslide could not move upward.

The calculation steps are described below.

1)Calculate the critical horizontal seismic acceleration k_c of the slope, that's the a_g value for which is F_s=1. In case of infinite slope k_c has the following form:

$$k_c = \frac{\frac{cb}{\cos\alpha} + \left[(W+Qb)\cos\alpha - U\right]\tan\varphi - (W+Qb)sen\alpha}{\frac{1}{2}W\cos\alpha}$$

2)Examine the recorded $a_g(t)$ values by the accelerogram using a reading step Δt equal to the recording one; the landslide motion starts at time t_0 for which is $a_g(t) \geq k_c$.
3)Calculate the landslide displacement through a double numerical integration of $a_r(t)=a_g(t)-k_c$, to get the following relations:

$$(1)\; s(t+\Delta t) = s(t) + v(t)\Delta t + \frac{2a_r(t) + a_r(t+\Delta t)}{6}\Delta t^2$$

$$(2)\; v(t+\Delta t) = v(t) + \frac{a_r(t) + a_r(t+\Delta t)}{2}\Delta t\;;$$

Naturally at time t=0 they are s(t)=0 and v(t)=0.
4)Multiply the calculated displacement, relative to a rigid block lying on a horizontal sliding plane, by a factor A as a function of the real shape of the sliding plane. In case of a planar surface, A has the following form:

$$A = \frac{\cos(\varphi - \alpha)}{\cos\varphi}$$

where φ is the angle of shear resistance acting along the sliding plane and α is the slope of the plane itself.

5)Apply the formula (1) and (2) to the next Δt intervals, through an iterative process until it will be verified the following condition:

$$v(t + \Delta t) = v(t) + \frac{a_r(t) + a_r(t + \Delta t)}{2}\Delta t = 0$$

6)Proceed reading the ag(t) values till the end of recording, repeating the steps 2) and 3) till when the condition ag(t)≥kc is verified.

As an alternative, displacement can be evaluated, in an approximately way, through empirical formulas, as the Jibson one (1993):

$$Log_{10} S_0 (cm)=1.460\ Log_{10} I_{arias} - 6.642\ a_{gcrit} + 1.546;$$

where I_{arias} is the Arias intensity and a_{gcrit} is the critical seismic acceleration, that is that one for which SF=1.

The cumulative displacement in the intervals where $a_g(t) \geq a_c$ can be used to have an estimation of the damage level caused by the seism to the structures lying on the slope based on the following table (Legg e Slosson, 1984):

Damage level	Cumulative displacement (cm)
Irrelevant	<0.5
Moderate	0.5-5
Strong	5-50
Severe	50-500
Catastrophic	>500

As to the maximum tollerable displacement, that's the displacement beyond which the slope has to be reguarded as unstable, it can refer to the folowing scheme(ASCE, 2002 e Wilson e Keefer, 1985):

Description	Max.displacement (cm)
Rock slope	2
Slope with engineering structures	5
Slope without engineering structures having a ductile behaviour (loose sand and gravel, N.C. clay)	15
Slope without engineering structures having a not ductile behaviour (dense sand and gravel, S.C. clay) where the peak shear strength is mobilised.	5
Slope without engineering structures having a not ductile behaviour (dense sand and gravel, S.C. clay) where the residual or ultimate shear strength is mobilised.	15